DYNAMIC CONSENSUS CONTROL OF LINEAR DISTRIBUTED MULTI-AGENT SYSTEMS

A PROJECT REPORT

submitted by

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to

the APJ Abdul Kalam Technological University
in partial fulfillment of the requirements for the award of the Degree

of

Master of Technology
in

Electrical and Electronics Engineering

with specialisation in

Industrial Instrumentation and Control

Department of Electrical and Electronics Engineering

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JUNE 2023

DECLARATION

I undersigned hereby declare that the project report entitled "**Dynamic Consensus Control of Linear Distributed Multi-agent Systems** ", submitted for partial fulfillment of the requirements for the award of degree of Master of Technology in Electrical and Electronics Engineering with specialisation in Industrial Instrumentation and Control, of the APJ Abdul Kalam Technological University, Kerala is a bonafide work done by me under the supervision of *Dr. Resmi R*, Project Guide, Assistant Professor, Department of Electrical and Electronics Engineering, *Prof. Amal A.*, Project Co-ordinator, Assistant Professor, Department of Electrical and Electronics Engineering, and *Prof. Sumayya Jaleel*, Assistant Professor, Department of Electrical and Electronics Engineering. This submission represents my ideas in my own words and where ideas or words of others have been included. I have adequately and accurately cited and referenced the original sources. I also declare that I have adhered to ethics of academic honesty and integrity and have not misrepresented or fabricated any data or idea or fact or source in my submission. I understand that any violation of the above will be a cause for disciplinary action by the institute and/or the University and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been obtained. This report has not been previously formed the basis for the award of any degree, diploma or similar title of any other University.

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CERTIFICATE

This is to certify that the report entitled " **Dynamic consensus control of linear distributed multi-agent systems** " submitted by **ANUGRAHA VINCENT** , **(Reg. No. TKM21EEII03)** of fourth semester to the APJ Abdul Kalam Technological University in partial fulfillment of the requirements for the award of the Degree of Master of Technology in Electrical and Electronics Engineering with specialisation in Industrial Instrumentation and Control, is a bonafide record of the project done by her under our guidance and supervision. This report in any form has not been submitted to any other University or Institute for any purpose.

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ACKNOWLEDGEMENT

A lot of effort and hard work has been put into this project in course of its presentation. However, it would not have been possible without the kind support and help of many individuals and other sources. I would like to extend my sincere thanks to all of them. I take this opportunity to express my deep sense of gratitude and sincere thanks to all who helped me to complete this project report successfully.

I thank God Almighty for paving my way throughout this project. I thank *Dr. Sabeena Beevi. K*, Associate Professor and Head, Department of Electrical and Electronics Engineering and *Prof. Shanavas T. N*, PG Co-ordinator, Department of Electrical and Electronics Engineering for their support and help. I express my sincere thanks to *Dr. Imthias Ahamed T. P*, Head of Centre for Artificial Intelligence, for his valuable help.

I am greatly thankful to my guide *Dr. Resmi R*, Assistant Professor, Department of Electrical and Electronics Engineering, for her supervision, and helpful suggestions. I am thankful to *Prof. Amal. A*, Project Co-ordinator, Assistant Professor, Department of Electrical and Electronics Engineering, and *Prof. Sumayya Jaeel*, Assistant Professor, Department of Electrical and Electronics Engineering, for the positive criticism and valuable comments.

Finally I thank my parents and friends who directly and indirectly contributed to the successful completion of my project work.

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ABSTRACT

Due to the recent advancements in the modern control theory, the technology for controlling a single system has reached a considerably matured phase. A single complex system when equivalently replaced by several simpler agents, greater benefits can be achieved. Multi-agent system (MAS) is defined as a loosely coupled structure composed of multiple agents, which interact each other to solve problems. With more research and observations done on the self organization and local interactions among various biological populations in nature, like the division of labour among ant colonies, formation of bird groups MASs emerged. MASs are widely used in spacecraft, automated highway systems, unmanned aerial vehicles, attitude synchronization of satellites etc. Cooperative applications of MAS include flocking, formation and consensus.

The first stage of the work analyses consensus problem which is an integral area of research under MAS. Static consensus control of MAS is studied using state feedback control. Considering the limitations of the state feedback controller in handling constraints, the control law is extended using an optimum control strategy utilizing a distributed model predictive controller (DMPC), which incorporates the constraints involved in the problem. Dynamic consensus control of linear distributed homogeneous agents with fixed communication topology is then studied utilizing the problem of vehicle platooning. The vehicle's state space model is derived from its lateral dynamics. The DMPC control is examined with and without constraints, with different sets of initial conditions. The results of MATLAB simulations demonstrate the effectiveness of the recommended control scheme to solve the benchmark problem of vehicle platooning.

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ABBREVIATIONS

LTI	Linear Time Invariant
MPC	Model Predict Control
DMPC	Distributed model predictive control
MAS	Multi-agent systems
PID	Proportional integral derivative
SOCP	Second-order cone programs
LQR	Linear quadratic regulator
ZOH	Zero order hold
ADC	Analog to digital conversion
ABS	Anti-lock braking system
QP	Quadratic program
NLP	Nonlinear program
UAV	Unmanned aerial vehicles

NOTATIONS

l_a	: distance from point C to front axle.
l_b	: distance from point C to rear axle.
I_z	: moment of inertia about Z axis.
$C_{\alpha a}$	: Cornering stiffness of combined front tires.
$C_{\alpha b}$	: Cornering stiffness of combined rear tires.
δ	: steering angle.
α_a	: slip angle at front tire.
α_b	: slip angle at rear tire.
θ	:Heading angle or yaw angle
S	:yaw rate
V_{LO}	:Longitudinal velocity
V_{LA}	:Lateral velocity
y	: Lateral position
f_a	: Front tire force
f_b	: Rear tire force
$\Re$	: Set of real numbers
N	: Set of positive integers
$\mathbb{A}$	: Adjacency matrix
$\mathbb{L}$	: Laplacian matrix
$\vee$	: Vertices in graph
E	: Edges in graph
a_{ij}	: Elements of adjacency matrices
l_{ij}	: Elements of Laplacian matrices
Q_i	: State of system with $\imath$ th agent
$\tilde{A}$	: System matrix

$\tilde{B}$	: Input matrix
u_i	: Control input
$\tilde{Y}$	: Output of the system
$\tilde{C}$	: Output matrix
$\mathbb{E}$	: Desired consensus value
C	: Reference point: Vehicle's center of gravity
δ	: Steering angle
β	: Slip angle
m	: Mass of the vehicle
U	: Control vector
$\mathbb{K}$	: State feedback gain matrix
M	: Control horizon
P	:Prediction horizon
k	: Sampling instant
$\tilde{A}_d$	: Discretized system matrix
$\tilde{B}_d$	: Discretized input matrix
$\tilde{C}_d$	: Discretized output matrix
$\Delta u_i(k)$	: Incremental control input
$\Delta Q_i(k)$	: Incremental state vector
A	: Augmented state matrix
B	: Augmented input matrix
C	: Augmented output matrix
$\mathbb{G}$	: Predicted state matrix in compact form
ϕ	: Predicted control matrix in compact form
F_{veh}	: Cost function of vehicle system
$\mathbb{C}_i\left(k\right)$	: Consensus point for $\imath$ th vehicle.

Chapter 1

INTRODUCTION

1.1 GENERAL BACKGROUND

Over the past few decades, modern control theory has advanced as a result of the rising technological industry, which is dependent on large engineering systems. During this rapid and persistent development, the technology for controlling a single system has become reasonably mature and has developed many effective tools, despite being higher dimensional and complex. Researchers began to investigate the idea of decentralized control which means breaking down a big system into smaller subsystems, each with its own controller, and then coordinating the controllers to attain a shared goal. This method was thought to be a good technique to improve the performance and robustness of control systems. Decentralized control theories have been used by researchers to multiagent systems (MAS), which are systems made up of several independent agents working together to accomplish a single objective. This approach, also known as multiagent control, soon gained acceptance in the field of control theory. Later on further analysis on this subject gave rise to a more efficient way of controlling interconnected multiple entities which is the distributed approach. Compared to centralised networks, distributed systems provide greater security, stability, and scalability. However, control is split equally throughout each node, unlike decentralised networks that employ clusters of smaller centralised servers. Recent years have witnessed a trend to use networked multiple autonomous agents to accomplish complex tasks arising from space-based applications, smart grids, intelligent transportation and machine learning.

Various biological populations in nature served as an inspiration for the emergence of MAS. Observing the interactions among several creatures in environment where groups of agents use specific interaction mechanisms at various levels to accomplish a common objective gave rise to MAS control. School of fish and bird flocks are common examples of MASs, which have attracted researchers from a wide range of fields, including physics, biology, and computer sciences. Figure 1.1 depicts a school of fish. They swim in groups rather than indi-

Figure 1.1: School of fish

vidually as they search for nourishment beneath the seas to avoid predators and find food more quickly. There will be more eyes on opponents when travelling in a group as opposed to as an individual agent. The same approach is used by flocks of birds to migrate over large distances together while keeping a pattern and spacing between them, which helps them escape predators. It also allows them to move together without getting lost.

Figure 1.2: Flock of birds.

Because of their ability to work in parallel, MASs can be used to address technical problems that are too difficult or time-consuming for a single agent to coordinate. A network of several collaborating cameras can be utilized to accomplish the goal instead of using a single camera to cover a broad region, for instance. This serve as a best application of MAS. In order to perform a shared job, these autonomous agents interact with one another to form the loosely connected structure referred to as MAS. A few of the traits of multi-agent systems include the following:

- Decentralization: These systems are usually distributed, which means that there isn't a single controller directing how each agent behaves. Instead, depending on information from its immediate surroundings, each agent acts independently.
- Heterogeneity: Agents may differ in terms of their actuators, sensing, or strategies for making decisions. Due of their distinctive characteristics they can complement one another and work together to achieve a shared objective.
- Autonomy: It is the ability of each agent in a system to behave freely in order to achieve its goals. Based on their experiences, agents can learn and alter their actions periodically.
- Scalability: These systems can be configured to scale to large number of agents, making it possible to employ them in applications ranging from traffic management to manufacturing to military operations.
- Interaction: To accomplish their group objectives, agents engage with one another in a cooperative or competitive manner. These interactions may be explicit, like through communication channels, or covert, such through the sharing of resources.

Interactions in MAS relate to the manner in which autonomous agents communicate, work together, compete with one another in order attain specific goals or collectively solve complex challenges. These interactions can take many forms and are vital to the overall behaviour and performance of the system. Interactions among MAS typically take the following forms:

- Communication: Agents can communicate information through several communication routes such as direct message passing, memory sharing, or broadcasting. Communication allows agents to exchange information, coordinate their activities, make agreements, and settle disagreements.
- Cooperation: It usually involves sharing of resources, division of duties, and synchronization of actions. Cooperative interactions can result in innovative patterns of behaviour,

enhanced efficiency, and the ability to tackle complicated issues that individual agents are incapable of handling alone.

- Coordination: To prevent disputes, synchronize their actions, and accomplish common goals, agents must coordinate their activities. The foundation of coordination mechanisms might be established norms and conventions, explicit communication, implicit signaling, negotiation procedures, or any combination of these.

- Competition: It is the way that different agents interact and struggle with each other as they try to reach their own goals in an integrated environment. In this situation, agents are smart entities that can understand their surroundings, make choices, and act based on their targets.

Each networked agent follows a defined structure or pattern of communication with respect to the connections among the agents, which is referred to as communication topology. It governs how agents are linked together and how information traverses between them. The communication architecture chosen can have a considerable influence on the system's performance, efficiency, scalability, and robustness. Commonly used interaction topologies are:

Figure 1.3: Centralized Topology of MAS

- Centralized Topology: There is a central agent or central communication node in this framework that serves as a focal point for all communication. All agents in the system connect with the central agent directly, which then transmits or distributes information to the agents. This layout facilitates coordination but can lead to a single point of failure and communication bottlenecks.

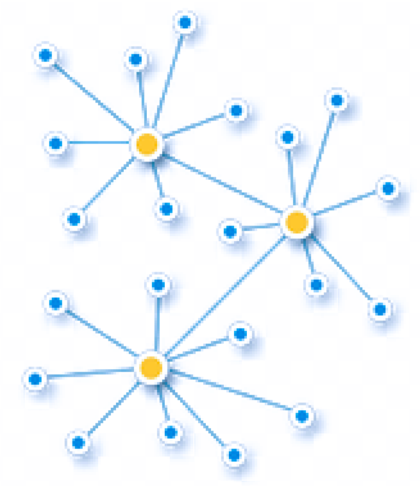


Figure 1.4: Decentralized Topology of MAS

- Decentralized Topology: Here agents communicate directly with each other since there is no central point. Each agent maintains its own set of communication links and trades information with its neighbours. However, there are layers of nodes, which is different. The secondary nodes (shown in yellow) are connected to the end nodes or blue nodes. Secondary nodes are linked to one another. Global coordination and synchronization might be difficult to establish.

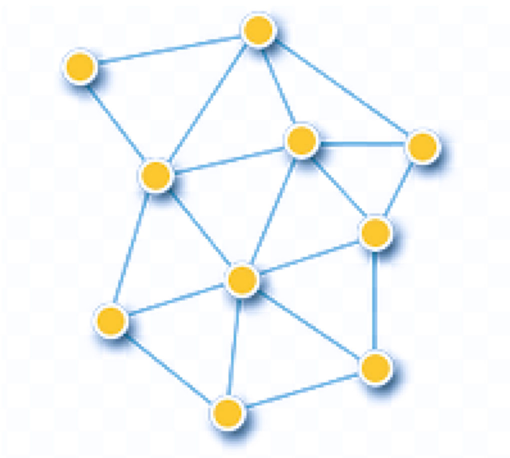


Figure 1.5: Distributed Topology of MAS

- Distributed topology: There is no central authority in distributed systems. Each dot in the diagram represents a node, or a network member. Each node is directly linked to every other node where each agent has equal power to one another.

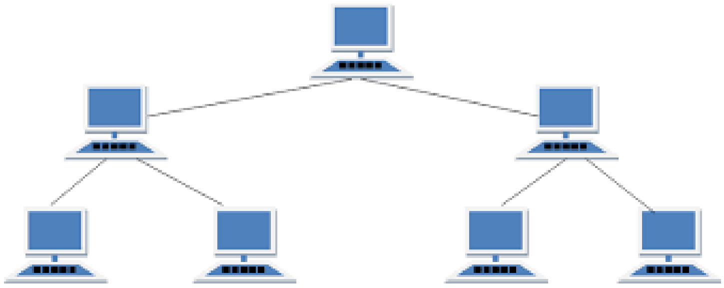


Figure 1.6: Hierarchical Topology of MAS

- Hierarchical Topology: Agents are organized into multiple levels or layers in a hierarchical structure. Communication takes place predominantly within each level, with higher-level agents in charge of gathering and transmitting information to lower-level agents. This design allows for efficient interaction and cooperation within each level, but it can add delays and overhead when information needs to be sent across levels.

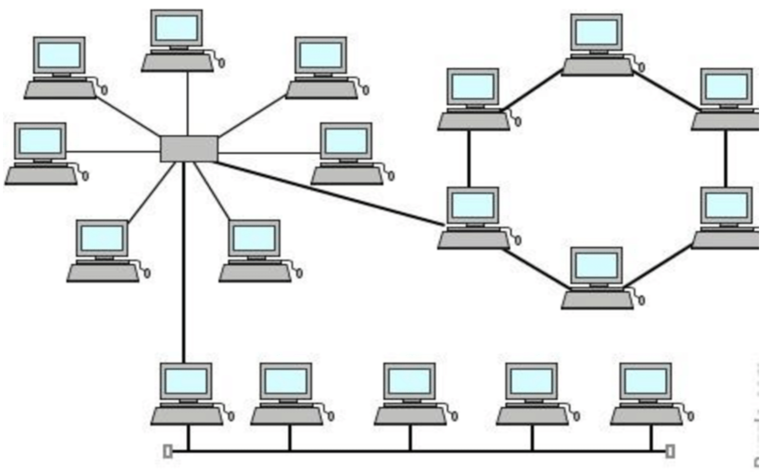


Figure 1.7: Hybrid control of MAS

- Hybrid Topology: To capitalize on their strengths, these topologies incorporate several communication patterns such as centralized, decentralized, or hierarchical. In sophisticated multi-agent systems, this enables more flexible and adaptive communication.

The communication topology chosen is determined by the MAS's unique requirements, restrictions, and dynamics, such as communication overhead, scalability, reliability, and the need for global coordination or local communication.

To depict the communication flow in linked MAS, network topologies for information sharing are modelled using graphs. Each node in the graph represents an agent. A graph can be classified as directed or undirected. An edge (i, j) in a directed graph (digraph) is an ordered pair of nodes that signals that information travels from agent i to agent j but not vice versa.

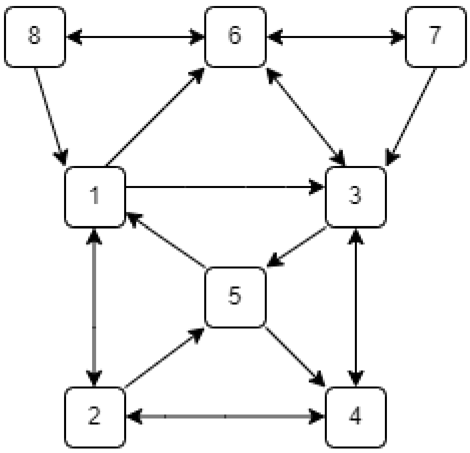


Figure 1.8: Directed graph

Figure 1.8 shows a MAS interconnection topology with a directed graph. An edge (i, j) is a pair of unordered nodes in an undirected graph that denotes the flow of information from agent i to agent j and vice versa. An undirected graph used to represent the MAS's connectivity architecture is shown in Figure 1.9.

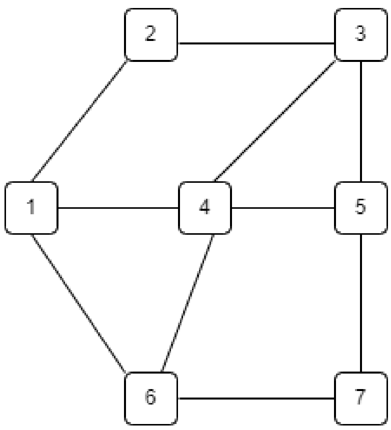


Figure 1.9: Undirected graph

Each agent communicate with its neighbours based on their communication topology in order to accomplish a particular task. For controlling multiple systems, the most commonly employed topologies are: the centralized and distributed. The former method is based on the assumption that a centralized location is available to control a large number of agents. This technique is simply a continuation of the traditional single agent based control method. Figure 1.3 depicts a centralized MAS. In the system, there is a central node or agent (yellow node) that has access to all of the other agents information. One may destroy the central controller in order to disrupt the MAS' connection. This results in the MAS being divided into independent agents, making interaction impossible. As a result, the fact that this system is vulnerable to attack or failure on the central node is one of its key weaknesses. On the other side, the information of every other agent must be shared by the central node. This would not be an issue in a small system, when there are few agents. This will place a significant strain on the central node, which is typically challenging to construct, in terms of communication and computing when the system size grows substantially. On the other hand, the distributed technique removes the necessity for a central controller at the expense of having a far more elaborate organisational structure.

An illustration of a distributed MAS is shown in Figure 1.5. A decentralized MAS has locally centralised agents, but a distributed MAS has no centralised agents at all, whether globally or locally. This is the main distinction between a distributed and a decentralised MAS. In case of decentralized system the network may be divided into a number of communities that can interact or communicate with one another through community nodes. Since the failure of an individual agent only affects the connectivity of its community and not the other sections of the network, this type of network structure is advantageous compared to a centralized one. Though both the cases have similarities and advantages over centralized one, when designing MAS, the distributed property is a crucial necessity. The basic goal of distributed control of a set of autonomous agents is to get every agent in the group to cooperate with one another across a distributed protocol. Here, the term "cooperative" relates to a deep bond between all of the group's members in which information exchange serves as the greatest significance. Flocking, formation control, trajectory tracking, and consensus control are a few examples of cooperative MAS applications. This work analyses consensus control of distributed MAS. Consensus in a network of agents is when the states of all agents reach the same level. The value at which all agents agree is known as the consensus value. The consensus algorithm outlines how the agents use their acquired knowledge to establish the agreement. If a consensus algorithm can be constructed in such a manner that each agent in the network only uses the knowledge of its respective neighbours, then the consensus method is distributed. Figure 1.10 depicts how the various agents interacted with one another to reach an agreement. Consensus can be static or

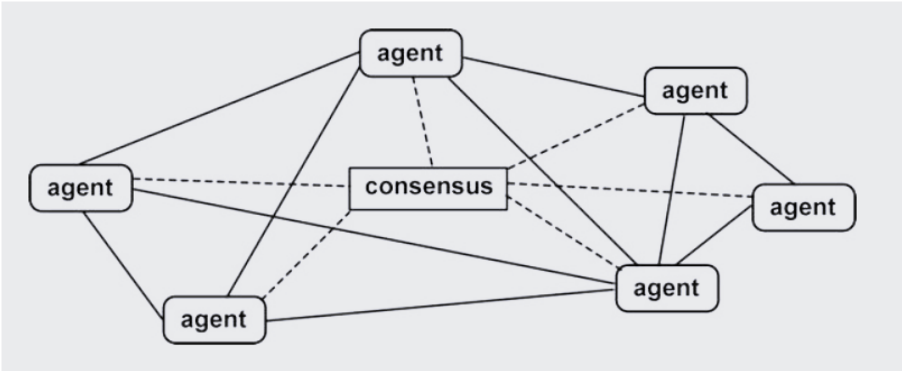


Figure 1.10: Consensus of Distributed Multi-agent systems

dynamic. Static consensus refers to the situation where agents aim to reach a consensus on a fixed value, often referred to as a set point or reference value, without considering dynamics or time-varying objectives. The goal is for all agents to agree on a common value, regardless of their initial states or individual dynamics. Dynamic consensus in a multi-agent system refers to the process of achieving agreement among agents not only on a fixed value but also on time-varying objectives or trajectories. Unlike static consensus, dynamic consensus takes into account the evolution of the system states over time and aims to synchronize the behaviors of agents accordingly. Primarily this work aims to achieve static consensus control, using state feedback controller. This control approach is then updated using a distributed model predictive controller to achieve dynamic consensus. Dynamic consensus of linear distributed homogeneous MAS where the agents reach an agreement on time varying trajectory is studied in this work using the benchmark problem of vehicle platooning. A group of five identical ground vehicles is taken as MAS. For developing the model of agents a dynamic bicycle model with linear tyres is considered and its state space representation is developed [1]. Based on the derived model four states: lateral position, velocity,yaw angle and yaw rate represents the system dynamics and the steering angle of vehicle represents the control input. Thus the highlights of this work is to achieve consensus of linear distributed MAS using different control strategies, also to study the mechanism of platoon formation of the vehicles.

1.2 OBJECTIVES

- Develop the state space model of a fourth order MAS .
- Propose suitable control strategies to control the four states under study, the position, lateral velocity, yaw angle and yaw rate based on the model derived, with the steering angle as control input.
- Analyse static consensus with the state feedback control.
- Apply the proposed DMPC algorithm to obtain dynamic consensus control of linear distributed MASs where the agents reach an agreement on time varying trajectory.
- Analyse DMPC control without constraints and also with constraints under two different sets of initial conditions.
- Verification of the proposed theory by the simulation of results on MAS, thereby examining the solution for the benchmark problem of platooning.

1.3 SCOPE

Interdisciplinary features of control theory, optimisation, artificial intelligence, game theory, network science, and distributed systems are all included in the scope of distributed control of multiagent systems. As research industry investigate new issues, create novel algorithms, and apply distributed control strategies to real-world problems, this discipline continues to progress. Applications include swarm robotics, distributed sensor networks, smart grids, robotics, unmanned aerial vehicles (UAVs), robotics, and transportation systems. The primary goal is on creating domain-specific control techniques and strategies that make use of the advantages of decentralised control in these particular areas. In the context of consensus control, MAS have a significant scope and play a crucial role. These systems encompass various consensus algorithms that determine how agents update their states or decisions based on the information exchanged with other agents. These algorithms define the convergence properties, stability, robustness, and performance characteristics of the consensus process. Examples of consensus algorithms include the average consensus algorithm, distributed averaging algorithms, and various variants of the consensus protocol.

This also addresses consensus control in dynamic environments where agents can join or leave the system, and the network topology can change over time. Adaptive consensus algorithms can be designed to handle such dynamics, allowing agents to adapt their behaviors and achieve consensus in the presence of uncertainties or agent mobility. Furthermore, optimization techniques can be employed to optimize the consensus process, such as selecting communication topologies, designing consensus protocols, or tuning control parameters. There are a wide range of optimisation methods in the literature which are still under study, some of them include the Fmincon optimisation, quadratic programming, primal-dual method, active set method. Extending current work to study the efficancy of these methods in another area of future scope realated to this study. By analysing all these control strategies the benchmark problem of Vehicle platooning is studied here, which refers to a technique where a group of vehicles travel closely together in a coordinated manner, typically using advanced technologies such as vehicle-to-vehicle (V2V) communication and automated driving systems. The main objective of platooning is to improve efficiency, safety, and sustainability in intelligent transportation.

1.4 ORGANISATION OF REPORT

The report is organized in 6 chapters. Chapter 1 titled by introduction includes general background, objectives and organisation of report. Chapter 2 surveys the literature review done on the phase 1 of the project work. Problem formulation along with consensus based control law is addressed in chapter 3. Chapter 4 titled by methodology incorporating the work done on the mathematical modelling of the system. Chapter 5 contains the control strategies used. Chapter 6 analyses the results obtained and chapter 7 is the conclusion.

Chapter 2

LITERATURE REVIEW

2.1 OVERVIEW

The study done for the analysis and formation of MAS control mechanisms is described in this chapter. On the basis of surveys, reviews, and other MAS-related publications, a thorough research is first undertaken on the characteristics, uses, kinds, and various control techniques of MAS. Section 2.2 comprehends the detailed review done on the developments in the study of MAS till date. Literature regarding the consensus control of MAS are examined and it is condensed in section 2.3. The selection of the MAS model and its mathematical modelling are the subject of a thorough investigation which is summarized in section 2.4. Analysis is also done on the control measures used on the system. Section 2.5 highlights the works done on state feedback control and DMPC. Since vehicle platoon formation is studied in this work, studies done on it is encapsulated in section 2.6. For the first phase of this project work, a mathematical model of ground vehicle is finalised. Next phase comprises of the implementation of control measures and optimisation algorithm adopted to obtain expected output. Further explanation of the literature review and its sources is provided herewith.

2.2 RECENT PROGRESS IN STUDY OF MAS

The Wright Brothers' first test flight in 1903 may be considered as the beginning of control system study and application in the last century. Control theory has increased in significance since then, resulting in navigation and guidance, fire-control mechanisms and automation. Another effect of improvements in this field was aerospace technology, which is based on vast engineering systems. Many relevant control tools like robust control,PID,adaptive control and non linear control have been developed throughout the rapid development of modern control theory. This has led to a considerably saturated state in the field of rising technology, that paved way for utilising the various advantages of replacing a single complicated system with a number of smaller, simpler ones. Thus an increase in demand for multi-agent control, particularly during the last two decades has been observed[2]. Nature has created a wide range of MASs in which groups of agents uses specific interaction methods at different levels to achieve a shared group objective. Common examples are swarms of bees, flock of birds, ant colonies, which have attracted scientists from various fields including information technology, mathematics, social science and physics. They may be used to resolve technical challenges that would be difficult or impossible to handle with a single agent due to their parallel features. Coordination and cooperation between these agents are necessary for the proper interaction based on an interaction topology. Also these features make them suitable for their distributed performance. These systems finds applications in many sectors like space science, intelligent transportation, robotics and power systems[3].

A survey on MAS clearly depicted more complete explanation of the agents definition, properties, and its formation to MAS along with its applications[4]. Based on this enquiries on MAS , a high-level comprehensive discussion regarding diverse aspects of MAS which helps newcomers to grasp basic concepts of MAS, study existing applications in multiple disciplines, the challenges in developing MAS, its evaluation and communication and the methods to study MAS performance were developed. They first provided a definition of agents and MAS and outlined their key features, then discussed the main applications and challenges of MAS while introducing references for further studies. Next, communications between agents and methods to analyze the effectiveness of an agent-based system are described. A taxonomy of the applications and challenges of MAS was provided in addition to the discussions based on various approaches used to evaluate the performance of agent systems. The most highlighted feature in these intelligent systems is the division of labour among each individuals making it a distributed pattern rather than a single authority. This could yield a cumulative effect on their goals which is more beneficial than a single system performing it. As a result controlling approaches in these systems is crucial which mainly depends on its collective functions. MASs frequently exhibits a variety of conventional group behaviours that are governed by a few basic communication rules. Examples include how microorganisms build patterns, pendulum clocks synchronise. Over the past three decades, a few representative mathematical or physical models have been put out to examine these collective behaviours statistically. The Boid model was developed by Reynolds to explain the group behaviour of bird flocks using three fundamental local rules: alignment, attraction, and repulsion [5]. Vicsek and his associates presented the well-known Vicsek model in 1995 using relatively simple local alignment criteria[6]. Based on the theory of infinite matrices products the dynamics of the linearized Vicsek model in analysed by Jadbabie et al at 2003 [7]. In 2008, Lü and his coworkers combined the laws of attraction and repulsion to improve the well-known Couzin-Levin model [8]. All of these models have taken the study of MAS interactions to another stage, investigating further control strategies. The two main Control measures include centralized and distributed control, both having its own possibilities and limitations. The complexity of the system, its behaviour and its goal determines the type of control to be applied. Owing to its reliability and efficiency while handling large systems distributed strategy is mainly used in studying MAS which has been developed from distributed computing [9]. This had laid the basis of consensus control which is one among the most popular collective activities of MAS, which also includes flocking and formation [10]. Out of these cooperative applications consensus gains more popularity in terms of its applications and is studied in this project work [11]. The research gaps analysed in consensus control is depicted in the next subsection.

2.3 CONSENSUS CONTROL OF MAS

Consensus: where all the agents in the system reach a final agreement upon on a shared objective is the research topic since 1974 through the studies of DeGroot and has gained more popularity with the distributed control of MAS[12]. Theoretical investigations of the consensus problem primarily focuses on simulating a model which has been mentioned in the works [5] [6][7][8]. They have extended their studies using the graph theory and matrix analysis on the consensus law. Though most of the works in this filed was mainly focused on fixed and directed topology. Later this was elaborated using Laplacian matrix on examining the consensus on first order integrator MAS by Olfati-Saber and Murray, both switching topologies and time delays are considered in their work. It suggests a distributed consensus strategy that makes sure each agent agrees even if the topology of the network changes over time.[13]. The consensus challenge of the second-degree integrator MASs is examined on the work[14] which mainly emphasis on the significance of communication topology, particularly the directed spanning tree, in attaining common objective within the system. In addition to it they studied the concept of consensus in the presence of communication delays. This work gives a thorough mathematical study of the consensus algorithm and proofs that it is stable in different delay situations.The development of the Laplacian matrix and graph theory turned to be a remarkable change in the study of consensus in terms of conceptual analysis which are the essential tools for future studies in this context.

Further examinations by various researchers has introduced different types of consensus control mechanisms based on constraints, switching topologies, time delays, consensus for continuous time and discrete MAS, leader following and group consensus which again has different branches to investigate and is explained on several literatures. Since works on leader following consensus are widely accepted in terms of both homogeneous and heterogeneous agents, many studies related to it provides ample knowledge especially on situations with switching topologies and delays with both first order as well as higher order systems. Also works on adaptive distributed control is provided to counteract with the switching topology conditions [15] [16] [17] [18] [19]. Later on works related to group consensus has gained more acceptance when control was focused on collaborative functions of agents. Distributed control of MAS has a great significance with group consensus as it is necessary to implement a proper control law for agents with different opinions to meet a common goal. This field is also extended to time delays, switching topologies, constraints and also on both homogeneous and heterogeneous agents with both directed and undirected communication links [20] [21] [22] [23] [24]. Based on triggering mechanisms in digital signals it has event triggering and time triggering applied to different models based on complexity using various computational algorithms [25] [3]. Also it is known that switching from continuous signal control to digital signal control can result in significant savings in terms of both the exchange of information and the resources allocated to computation [26] [27]. Since majority of the existing works mainly rely on homogeneous agents with fixed topology and directed communication network, recent trends mainly involves works on heterogeneous agents with dynamic consensus. As this could lead to more complex systems suitable control measures should be design to incorporate it. Also dynamic consensus using undirected topology with reduced computational techniques by distributed control is another interesting area comprising more possibilities to explore. Suitable results on these works would help to extend its applications on robotics, vehicle platooning and various intelligent systems.

2.4 MATHEMATICAL MODELLING OF MAS

Since vehicle platooning problem is examined in this work the selection of an accurate but simple model is crucial to develop an efficient control mechanism. Several works have been studied and finally arrived at a conclusion by choosing a ground vehicle as the linear agent and its state space model is developed by using the linear 3-DOF bicycle model. The use of bicycle models has been observed for a long time in trajectory control and other vehicular applications [28] [29]. Reducing computational burden and complexity in the system are the mains reasons for selecting this model which considers different assumptions in order to make a simple model without compromising its accuracy. Considering these possibilities this bicycle model was relevant from the beginning, when investigations for vehicle steering, stability and safety came into existence [30] [31]. Both linear and non linear models are used for vehicular applications but former gained popularity only in the recent years and still a lot more works has to be done on it to attain efficient results. Most of the existing surveys present linear models because analysis of system performance in context of various spacing strategies and communication patterns can be complicated by nonlinear models of vehicle dynamics [32]. This is how linear models are so popular. Single- and double-integrator systems are the simplest and more frequently used. This has a demerit as it ignores some aspects of vehicle dynamics that could affect system performance in real-time tests. Hence higher order linear systems are to be investigated for further examinations which could be more beneficial in reducing complexity as compared to non linear systems. To increase the precision of the tyre force, a simplified piecewise tyre model is proposed in [33]. This model makes advantage of the wheel rolling speed data from the real car, which is commonly used in Anti-lock Braking System (ABS) or other active safety measures. ABS is a safety measure, its primary purpose is to prevent the wheels from locking up during sudden or emergency braking situations, which helps the driver maintain control and steer the vehicle while coming to a stop. Simulation outcomes indicate that the recommended system is better as compared to the 14-DoF whole car model and can be used for chassis control. Vehicle Dynamics and Control by R.Rajamani provides a comprehensive coverage of vehicle control systems and the dynamic models used in the development of these control systems. Vehicle's longitudinal, lateral and roll dynamics along with the tire dynamics, both modelling and control aspects are clearly stated in this which helps researchers to a great extent [1]. Both lateral as well as longitudinal dynamics are commonly used in controlling vehicles. For platooning most prevalent dynamics in the literature is longitudinal but recent works shows many applications of the lateral dynamics [34] [35]. Moreover the type of dynamics used mainly depends on the platooning aspects and kind of application. To maintain the desirable platoon formation and reaping the benefits of reduced air resistance, longitudinal dynamics, such as speed control and spacing control, are crucial. Although there have been advancements in lateral dynamics, such as lane-keeping and cooperative lane changes, these technologies are not as widely implemented and adopted as longitudinal dynamics [36] [37]. A research gap is identified in the vehicle lateral dynamics for further advancements so this work mainly focuses on it by using steering control for achieving platoon formation.

2.5 CONTROL STRATEGIES

Although the research of MAS control is significantly important today, another fascinating but more crucial area within this discipline is the study of controller design for these multiple systems. Earlier works included the use of algebraic graph theory and Laplacian matrix with single integrator and double integrator dynamics [13]. Both centralized and distributed approaches are used in consensus control of MAS [38] [39]. If the number of agents in the system increases, practically it is inefficient to deal with a single central controller as it would decrease the communication performance and it could even worsen the situation in case of any disturbances. As a result practical applications mainly rely on distributed control mechanisms. Based on this work both static and dynamic consensus control techniques has to be investigated. Several works under static consensus are found in studies which mainly uses PID and linear consensus protocols. Also the choice of controller depends on the specific requirements of the system, the network topology, the desired level of decentralization, and the desired performance objectives. The goal is to design controllers that achieve consensus efficiently, robustly, and with minimal communication overhead among the agents. Research works regarding control methods like proportional derivative longitudinal controller and PID control are profoundly used [40]. Platooning problem under dynamic consensus requires more advanced controller, though PID and other robust control mechanisms like sliding mode control is observed in studies. The latter one mainly focuses on non linear systems [41]. Another existing control methods in observing dynamic consensus is the Adaptive control techniques. They are used when the agents' dynamics or interactions are uncertain or subject to changes. These controllers adapt their parameters over time to handle system uncertainties and achieve consensus under changing conditions. Since the control strategies studied in literature exhibits stability issues, one of the most widely accepted control technique, the state feedback control is taken under study in this work. Here the static consensus control is attained with the consensus algorithm using a state feedback approach.

Although simulations proves that the feedback controller aids in achieving the main objective of work, taking into account the chances of numerous constraints as we handle a system comprising of vehicle subsystems recent trends in control utilizes model predictive controller (MPC) which would be more realistically practicable when handling actuator in-capabilities [42]. Due to the fact that MPC design combines optimisation and the feedback mechanism, the inherent features of feedback can, in part, offer some degree of robustness against extraneous disturbances [43]. MPC uses the receding horizon method which computes the control parameters at each time interval and updates its information based on reference value. Its minimizes the square deviation caused by the current state values with the reference thereby maintaining the position in vehicle platooning and also constant velocity between each vehicles in platoon. MPC control algorithm includes an objective function and its main motive is to optimise the objective function on each computation interval which is the main reason why it is referred to as an optimization control strategy [44]. Majority of the works related to this study involves the use of distributed MPC for handling constraints, computational techniques for time delays with triggering mechanisms and also to deal with dynamic consensus [45] [46]. Constrained control using DMPC under consensus control strategy can be done with many optimisation techniques like Fmincon optimisation, convex optimisation, dual decomposition and second order cone programming(socp), here it is done with the constrained DMPC consensus algorithm which reduces the complexity and time for computation in MAS [47]. The constraints are imposed on the states and input of the system which shows the improvements in system performance while achieving vehicle platoon formation.

2.6 VEHICLE PLATOONING

Self-driving automobiles are prime examples of MAS. This technology has gained popularity recently due to its expanding market value on a worldwide scale and is essential for intelligent transportation. The most difficult challenge in the transportation sector due to rising automation is the management of these interconnected vehicles. Among the diverse applications and control of these vehicles, platooning has become a game changer. As a result it is now considered to be a more relevant area of attention for future studies [48] [49]. In a group of moving vehicles, the first vehicle generates a slipstream, or a wake of air, which lowers air resistance or drag. The following vehicles can benefit from this decreased drag by strategically positioning themselves in the slipstream, a process known as drafting. The following vehicles suffer less aerodynamic resistance when they follow the leading vehicle, which reduces energy consumption and boosts fuel efficiency. Advanced vehicle-to-vehicle communication systems are frequently used during platooning to synchronise the movements of the vehicles. The cars in a platoon can maintain a constant speed and spacing by coordinating acceleration, deceleration, and braking. By minimising abrupt speed changes and airflow disruption, this synchronisation improves the aerodynamic performance of the entire platoon. Platooning allows networked autonomous cars to maintain a desired spacing with their neighbours while moving at the same velocity, thereby relieving traffic congestion, reducing air pollution, and ensuring road safety [50]. Thus through automated highway systems, it improves road capacity, encourages safe driving behaviour, and helps cars become more aerodynamically efficient [51]. Platooning provides the aforementioned benefits, which have been empirically proven and demonstrated in the literature where an adaptive cruise control is being investigated and validations are carried out on four Infiniti M56s automobiles [52].

The Prometheus Project in Europe and the PATH project were the first steps in the research of vehicle platooning [34]. Cooperative adaptive cruise control, a new sector was evolved gradually from the traditional adaptive cruise control, in this domain related to these studies which contributes a lot of advancements and still it remains to be an emerging technology [52] [53] [35]. Later, many levels of analysis have been performed in this domain, and almost all of them have primarily concentrated on the use of longitudinal dynamics to align the vehicles in a formation utilising spacing policies. Only when studies extended to lateral dynamics lane keeping control came into light extensively [48] [36]. Combination of both the longitudinal and lateral dynamics can be seen in literature [54] [55]. Distributed control forms the basis of this study as more benefits can be reaped using a decentralized system. Vehicle-to-vehicle communication enables each vehicle in the platoon to update its current states through its onboard sensors and computation techniques thereby achieving consensus. Both leader following or group consensus can be applied into this problem but as majority of the research work focuses on latter one there arises a need to explore the benefits of former, as a result current research trend follows the platooning under group consensus in a distributed network using lateral dynamics [49] [50] [37]. While moving on to heterogeneous agents the complexity rises so mainly homogeneous MAS is concentrated primarily. Thus this work includes the platooning of linear homogeneous cars with distributed consensus strategy using a undirected communication topology.

2.7 RESEARCH GAPS IDENTIFIED

Based on the studies mentioned above some gaps are identified on the analysis of this work. Further studies on these gaps could generate various advancements in this field.

- In spite of all the countless works related to MAS, there arises an urge to conduct more works on dynamic consensus control of distributed linear MAS using a fourth order system. Since all the existing works works focuses on using single or double integrator dynamics, a fourth order system with lateral vehicle dynamics is studied in this work.
- The benchmark problem of platooning is still an emerging sector both industrially as well as analytically. Especially the applications of lateral dynamics using a distributed consensus algorithm rather than the prevalent leader following method needs greater investigations.
- Platoon formation under dynamic consensus control using distributed control techniques are yet to be examined more by using MPC since most of the existing works are related with matrix theory.
- Distributed MPC for a fourth order system are not commonly observed in the existing studies.So in order to reduce the computational burden and complexity while using higher order systems distributed MPC is more beneficial.

2.8 SUMMARY

This chapter discussed previous works related to consensus control of distributed MAS. The research gaps for each stages of the project work is studied and identified. Linear Homogeneous MAS with lateral dynamics is taken under study for further investigations of the research gap identified.

Chapter 3

PRELIMINARIES AND PROBLEM FORMULATION

3.1 OVERVIEW

This section gives the detailed idea of the basic notations and representations of MAS required to develop the control algorithm. Also the interaction topology of interconnected vehicles and main formulations of consensus control algorithm is described. Section 2.3 gives the graph theory concepts that are used to portray the communications between the networked vehicles. Section 3.3 discusses the flow of consensus control protocol for both static and dynamic consensus for platooning applications. The communication topology and laplacian matrix between the five vehicles in platoon is also depicted here.

3.2 GRAPH THEORY

Graph theory plays a fundamental role in the study of MAS. It provides a mathematical framework for modeling the interactions and relationships among agents and analyzing their collective behavior. In MASs, agents can be represented as nodes or vertices in a graph, and the relationships between agents are represented by edges connecting the nodes. The graph structure captures the connectivity and communication patterns among agents. Also it helps to characterize the network topology of the system. The arrangement of nodes and edges in the graph determines how agents are connected and influences the flow of information and coordination among them. In MAS, topology refers to the structure or arrangement of the communication links between the agents. It determines how agents interact and exchange information with each other. There are two main types of topologies: directed and undirected. The mathematical notations used in this work can be represented as: $\Re$ denotes the set of real numbers, **N** is the set of positive integers and $\Re^{s\times t}$ is the set of real matrices of order $s \times t$. The $\imath$ th agent is denoted by subscript $\imath$. Adjacency matrix is given by $\mathbb{A}$. Laplacian matrix is denoted as $\mathbb{L}$. Information exchange between the vehicles are done with the help of representations called graphs. Each interconnected vehicles are taken as nodes or vertices in the graph given by set $\vee$=$\{1, 2, ...N\}$ and the interconnections are given as edges with set $E = \{\vee \times \vee\}\,;\;\; E \subseteq \{(i,j)\,\forall i,\;\; j\varepsilon\vee,\;\; i \neq j\}$. The communication topology of the MAS is represented using a fixed, undirected graph where information flow occurs on both directions.Two fundamental matrices in graph theory are the adjacency matrix expressed as $\mathbb{A}$= $[\boldsymbol{a_{ij}}]\;\varepsilon\;\Re^{N\times N}$: $\imath$, $\jmath\;\varepsilon\;\vee$ and the value of a_{ij} is 1 if any information transmission takes place between agent $\imath$ and $\jmath$ and the edge set has an edge from node $\imath$ to $\jmath$ otherwise the value of a_{ij} is 0. The Laplacian matrix is generated from the adjacency matrix denoted as $\mathbb{L} = [l_{ij}]\;\varepsilon\;\Re^{N\times N}$, where $l_{ij} = \{degree\,(v_i)\,;\;\; if(i == j)\}$ or $l_{ij} = -1$; if $(i \neq j)$, where v_i represents the vertices v_1,....,v_n. The Laplacian matrix for the given interaction topology is given as: $\mathbb{L}$= $\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & -0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$

3.3 CONSENSUS CONTROL LAW

Consensus control laws in MAS play a crucial role in achieving cooperation, coordination, and agreement among autonomous agents, enabling them to work together effectively towards common goals. One well-known consensus algorithm studied here is the distributed average consensus algorithm, which ensures that the agents' states converge to the average of their initial states. The algorithm can be summarized as follows. Each agent initializes its state to a value. In each iteration, the agents communicate with their neighbors and update their states by taking the average of their current state and the states received from neighbors. The algorithm continues until the agents' states converge within a desired value. This control law is implemented to achieve static consensus of the MAS.

3.3.1 Static Consensus

Static consensus control deals with achieving consensus among a group of agents in a static or fixed environment. The goal is for all agents to agree on a specific value or state. The agents continuously exchange information with their neighbors and update their states based on the received information. The consensus algorithm used typically involves averaging the agent's state with the states of its neighbors. The states of the agents converge towards a common value, representing the consensus. Static consensus control is applicable in scenarios where the environment and system parameters remain unchanged over time. A network of distributed homogeneous agents with undirected network topology is considered with each agent of order 'n' and having 's' inputs and 'p' outputs. The system dynamics is given as

$$\dot{Q}_i = \tilde{A}Q_i\left(t\right) + \tilde{B}U_i\left(t\right), i\varepsilon\vee \tag{3.1}$$

$$\tilde{Y} = \tilde{C}Q_i\left(t\right) \tag{3.2}$$

where $\tilde{A}$ is the system matrix of order $\Re^{n\times n}$ and $\tilde{B}$ is input matrix of order $\Re^{n\times s}$ and $Q_i\left(t\right)\varepsilon\,\Re^{n}$ is the state of the system with agent $\imath$ and $u_i\left(t\right)\varepsilon\,\Re^{s\times}$ is the control input. $\tilde{Y}\,\varepsilon\,\Re^{p}$ is the output of the system with output matrix $\tilde{C}$ of order $\Re^{n\times p}$. For the given system, standard consensus control protocol is defined as

$$u_i = -\sum_{j=1}^{n} a_{ij}\left[Q_i - Q_j\right], i\varepsilon\vee \tag{3.3}$$

where a_{ij} constitutes the elements of the adjacency matrix $\mathbb{A}$ in the graph $\mathfrak{g}$. The closed loop dynamics of the system described in equations (3.1) and (3.2) can be expressed in a structured matrix as

$$Q\dot{\left(t\right)} = -\mathbb{L}Q\left(t\right) \tag{3.4}$$

where $Q\left(t\right) = \left[Q_1\left(t\right), Q_2\left(t\right)..., Q_N\left(t\right)\right]^T$ is the state vector and $\mathbb{L}$ is the Laplacian matrix. The MAS achieves static consensus under control law (3.3) only if

$$lim_{t\to\infty}\left|Q_{i(t)} - Q_{j(t)}\right| \to 0 \tag{3.5}$$

Static consensus is applied in distributed control systems for coordinated control and synchronization among multiple agents or controllers. In scenarios involving multiple robots with fixed connectivity, static consensus can be used to coordinate their movements or actions to achieve common goals.

3.3.2 Dynamic Consensus

Dynamic consensus control considers time-varying or dynamic environments. The objective is to achieve consensus among the agents while adapting to changing conditions. The agents need to continuously update their states as the environment evolves. The algorithm used for dynamic consensus control takes into account the time-varying nature of the system and incorporates the dynamics of the agents and the environment. The agents exchange information with their neighbors, updating their states iteratively based on the received information. The states of the agents continuously evolve over time, and the goal is to maintain consensus despite dynamic factors such as changes in reference signals, disturbances, or the presence of new agents. This consensus control is used in the study of benchmark problem of vehicle platoon formation. The system is said to achieve dynamic consensus under control law (3.3) if

$$lim_{t\to\infty}\left|Q_{i(t)}-Q_{\jmath(t)}\right|\to\mathbb{E};\forall Q_i\left(0\right);\imath,\jmath=1,2,...,N. \tag{3.6}$$

The constant $\mathbb{E}$ is the desired spacing of vehicles in platoon or the final state value which should be achieved by all the agents. State trajectories of agents arising from different points are found to be converging to a common point of agreement thereby attaining control'is depicted in Figure 3.1. Dynamic consensus of MAS finds numerous applications across various fields. In robotics and autonomous vehicle systems, dynamic consensus is essential for agents to cooperatively navigate, avoid collisions, and maintain formation control while adapting to dynamic obstacles or changes in the environment. Applications in distributed control systems includes power grids, traffic control, and industrial automation.

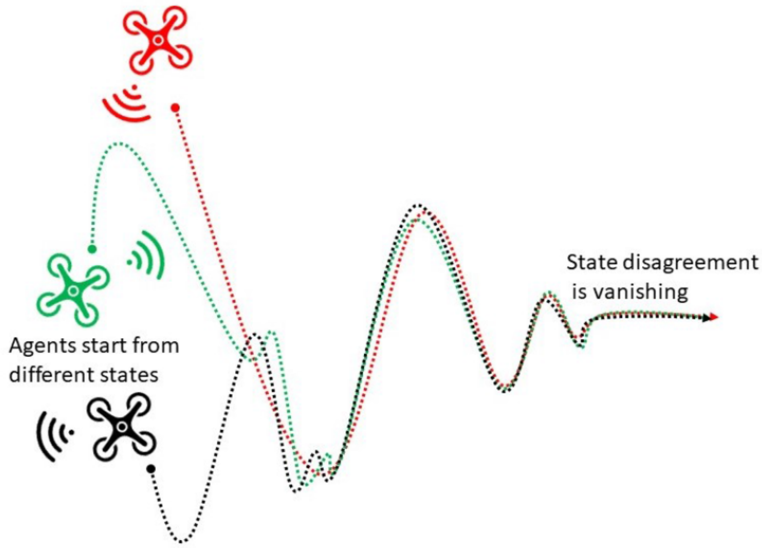


Figure 3.1: Consensus of MAS

3.4 SUMMARY

The basic knowledge of graph theory on consensus control of MAS is provided in this section. The idea behind using the conventional consensus strategy is exhibited. Both static and dynamic consensus control is described. Also the idea of how the multiple agents works using both the static and dynamic consensus control algorithms is emphasised in this section.

Chapter 4

MODELLING

4.1 OVERVIEW

This chapter discusses about the design concepts needed for developing the mathematical model of the system. Based on the problem identification and preliminary studies on previous sections a suitable MAS is selected which is a ground vehicle and developed its state space representation with the equations of motion which is described on section 4.2. The graphical representation of the system under study is depicted on Figure 4.1. The lateral and yaw dynamics considered to derive the equations of motion along with the linear tire model is presented in the below section.

4.2 MATHEMATICAL MODEL

The mathematical model of four wheeled ground vehicle is developed considering the lateral vehicle dynamics. The following assumptions are considered for modelling the system as a dynamic bicycle model with linear tires:

1. The first important and the most obvious one is the bicycle assumption in which the wheels of front and rear axle are lumped together into one front and rear wheel.

2. Reference point C is the vehicle's center of gravity which is needed in order to be able to apply the laws of motion so that the centre of gravity principle and angular momentum principle is applied. Also the vehicle mass is represented by the parameter "m" and the moment of inertia about Z axis is given by "I_Z" where z refers to the vehicle coordinate system but here we an other assumption pointed below.

3. Motion of the bicycle model happens only in the X-Y plane.

4. Motion of vehicle is described by two planar components: longitudinal velocity and lateral velocity and one angular speed which is yaw rate denoted by "S".

5. Another important assumption is that the vehicle is considered as a rigid body whose dynamics are determined by fundamental laws of motion which can also be referred to as the lateral and yaw motion that are determined by the forces, which in this case called as the tire forces that act on the front and rear tire of the vehicles.

6. The point of attack of the front wheel tire force is called as point 'a' and that of rear wheel tire force as point 'b'. The distance between a and C is denoted as l_a and the distance from C to b is l_b. The sum of l_a and l_b is the wheel base of vehicle. Assume that only lateral tire forces are acting on the front and rear tire of the vehicles. The forces f_a and f_b are perpendicular to the respective tires center line and these lateral tire forces are generated by a linear tire model. The parameters $C_{\alpha a}$ and $C_{\alpha b}$ represents the cornering stiffness of the combined front tires and combined rear tires respectively. The cornering stiffness of a vehicle refers to its ability to resist lateral forces and maintain stability while going around a corner. It is a measure of the vehicle's tire's resistance to deformation during cornering.

7. For derivations assume that steering angle δ is very small such that $sin\delta \approx \delta; cos\delta \approx 1; tan\delta \approx \delta$.

8. Longitudinal velocity refers to the component of a vehicle's velocity that is parallel to its direction of motion. It represents the speed at which the vehicle is moving forward or backward along its longitudinal axis. When considering lateral dynamics in a vehicle that is in longitudinal motion (moving in a straight line), the longitudinal velocity V_{LO} is constant. So the laws of motion is applied only for the lateral and yaw motion.

With all these assumptions the laws of motion is set up. For this the coordinate of inertial frame and those of vehicle frame is made use of. In vehicle dynamics the coordinates of an inertial frame and a vehicle frame refer to the reference systems used to describe the motion and orientation of a vehicle. The vehicle frame, also known as the body-fixed frame, is a reference frame attached to the vehicle itself. It moves and rotates with the vehicle's motion, allowing for a description of the vehicle's position, orientation, and dynamics from the perspective of the vehicle. The inertial frame is a reference frame that remains at rest or moves with a constant velocity in a straight line. In the context of vehicle dynamics, an inertial frame is often considered as an external frame of reference fixed to the Earth or any other non-accelerating object. It provides a stable reference for measuring the motion of the vehicle without considering the vehicle's own movements. The yaw angle or heading angle of the vehicle is represented by θ

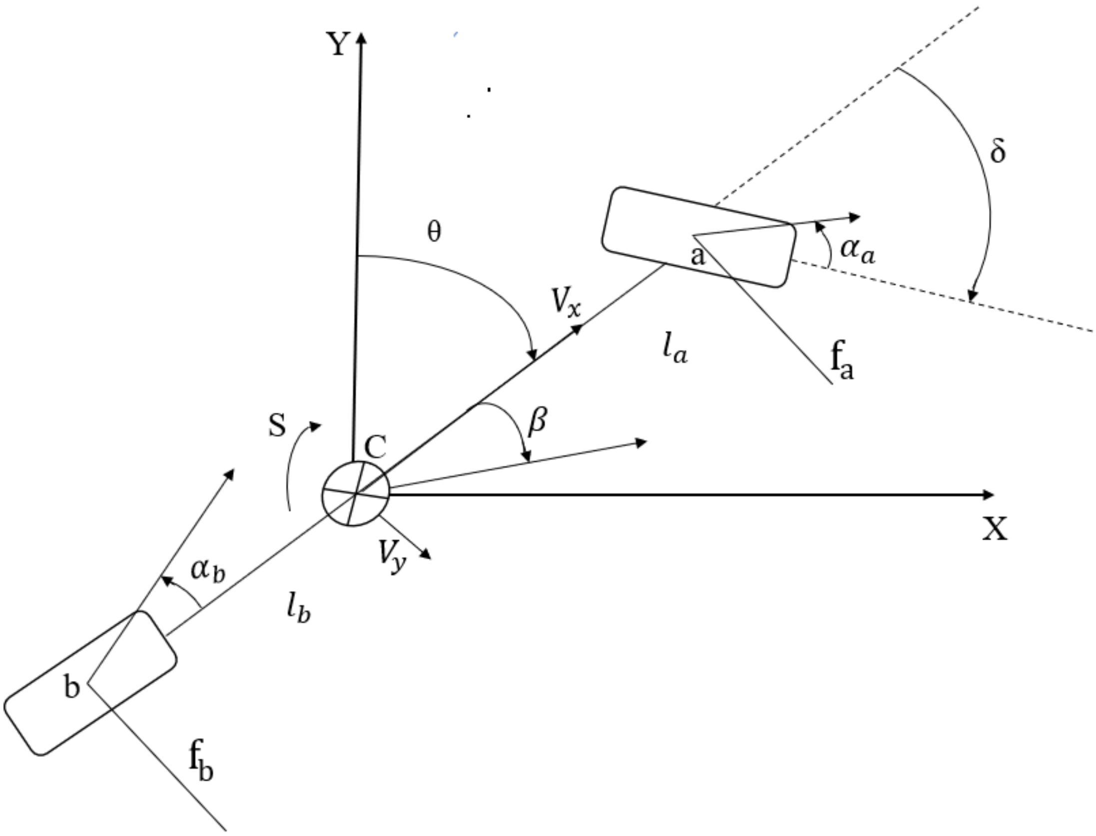


Figure 4.1: Dynamic bicycle model with linear tyres

and the yaw rate or angular speed is S. The slip angle of the vehicle which is the angle between the motion direction and vehicle orientation is β. Also α_a and α_b are the tire side slip angle of front and rear tires respectively. The steering angle δ is taken as the control input. The steering angle of a vehicle refers to the angle between the direction the front wheels are pointing and the longitudinal axis of the vehicle. It represents the degree to which the wheels are turned, either to the left or right, as controlled by the driver or an automated steering system.

4.2.1 Lateral Dynamics

By center of gravity principle the product of mass of vehicle and acceleration in Y direction in inertial system is given as the sum of all forces in Y direction that act anywhere on the vehicle.

This can be represented as: $m * \tilde{a}_Y^{(X,Y)} = \sum_i f_{yi} = f_b + f_a cos\delta \approx f_a + f_b$. Only two forces are acting on vehicle, the front and rear tire force. Y component of rear tire force is the full tire force and the Y component of front tire force is given as $F_a cos\delta$. Since vehicle frame is not an inertial frame, acceleration in Y direction in inertial frame is given by time derivative of the lateral velocity plus the centripetal acceleration of the wheel: $\tilde{a}_Y^{(X,Y)} = \dot{V}_{LA} + (S * V_{LO})$

$$m * \left[\dot{V}_{LA} + (S * V_{LO})\right] = f_a + f_b \tag{4.1}$$

4.2.2 Yaw Dynamics

By angular momentum principle we take the product of moment of inertia about Z axis and derivative of yaw rate S is equal to sum of all moment about center of gravity which in turn equals to two moments generated by the rear and front tire forces where the rear tire force has the lever arm l_b and it turns clockwise about point C so we have a minus sign and the front tire force has a lever arm l_a and only the component $l_a cos\delta$ turns clockwise in this case. So it has positive sign and the other component of front tire force along X axis has no lever arm so it drops out of equation.

$$I_Z * \dot{S} = -l_b f_b + l_a f_a \tag{4.2}$$

In order to complete the model, tire forces are to be derived from equation (4.1) and equation (4.2). For linear tire model, tire forces are proportional to the corresponding slip angles with the proportional constant being the cornering stiffness of the front and rear wheels ($C_{\alpha,a}; C_{\alpha,b}$). At this point we have to pay close attention to the sign we are using and sign depends on the direction we have assumed to the tire forces.

4.2.3 Linear tire model

$$f_a = -C_{\alpha a}\alpha_a \tag{4.3}$$

$$f_b = -C_{\alpha b}\alpha_b \tag{4.4}$$

To determine the slip angles at the front and rear tire,we need to look at the velocity geometry at points a and b. Here rear tire force is actually the easier case because the tangent of slip angle α_b is given as the ratio between velocity component of point B in Y direction and X direction.

$\alpha_b \approx tan\alpha_b = V_{B,Y} \div V_{B,X}$

$\alpha_a \approx tan\alpha_a = V_{A,\eta} \div V_{A,\xi}$

For front tire look at the velocity components in the direction of wheel coordinates η and ξ. From the laws of kinematics of rigid body of vehicle, velocity components in X and Y direction for point B and point A in terms of lateral and longitudinal velocity and yaw rate.

$V_{A,X} = V_{lon}; V_{A,Y} = V_{LA} + Sl_a$

$V_{B,X} = V_{lon}; V_{B,Y} = V_{lat} - Sl_b$

Using co-ordinate transformation of $(X, Y) \to (\xi, \eta)$ $V_{A,\xi} = V_{A,X}cos\delta + V_{A,Y}sin\delta$

$V_{A,\eta} = -V_{A,X}sin\delta + V_{A,Y}cos\delta$

Apply steering angle approximations as $sin\delta \approx \delta; cos\delta \approx 1; tan\delta \approx \delta.$

From further substitutions and calculations done with the above equations the front and rear tire force equations are obtained as follows. Also we assume that lateral velocity and yaw rate is small as compared to the longitudinal velocity, especially if it is multiplied with a small steering angleδ we obtain the simplified expression as :

$$f_a = C_{\alpha a}\delta - C_{\alpha a}\left(\frac{V_{LA} + Sl_a}{V_{LO}}\right) \tag{4.5}$$

$$f_b = -C_{\alpha b}\left(\frac{V_{LA} - Sl_b}{V_{LO}}\right) \tag{4.6}$$

Substituting equations (4.5) and (4.6) in equations (4.1) and (4.2) the state space model for an i^{th} agent is obtained as:

$$\begin{bmatrix} \dot{y}_i \\ \dot{V}_{LAi} \\ \dot{\theta}_i \\ \dot{S}_i \end{bmatrix} = \begin{bmatrix} 0 & 1 & V_{LO} & 0 \\ 0 & \frac{-2C_{\alpha a}+2C_{\alpha b}}{mV_{LO}} & 0 & \frac{-2l_aC_{\alpha a}+2l_bC_{\alpha b}}{mV_{LO}} - V_{LO} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-2l_aC_{\alpha a}+2l_bC_{\alpha b}}{I_ZV_{LO}} & 0 & \frac{-2l_a^2C_{\alpha a}+2l_b^2C_{\alpha b}}{I_ZV_{LO}} \end{bmatrix} \begin{bmatrix} y_i \\ V_{LAi} \\ \theta_i \\ S_i \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2C_{\alpha a}}{m} \\ 0 \\ \frac{2l_aC_{\alpha a}}{I_Z} \end{bmatrix} \delta_i \tag{4.7}$$

The four states being lateral position (y), velocity (V_{LA}), yaw angle (θ) and yaw rate (S) with steering angle (δ) as control input [1].

Table 4.1 gives the values of approximate vehicle parameters obtained from the study for further control investigations. These values are substituted onto the derived state space model which in turn yields the state space matrices of the system. These informations are useful in computing the consensus algorithm for each agent in the MAS to attain their consensus control both static and dynmaic.

Table 4.1: Vehicle Parameters

Parameters	***Description***	***Value***	***unit***
m	Mass of the vehicle	1500	Kg
l_a	Front axle length	1.2	m
l_b	Rear axle length	1.3	m
$\tilde{a}$	Acceleration due to gravity	9.8	m/s^2
$C_{\alpha a}$	Front wheel cornering stiffness coefficient	13500 0	N/rad
$C_{\alpha b}$	Rear wheel cornering stiffness coefficient	95000	N/rad
I_Z	Yaw inertia	2500	Kgm^2
V_{LO}	Longitudinal velocity	10	m/s

4.3 SUMMARY

Taking into account all of the assumptions established throughout the modelling process, dynamic bicycle model with linear tires for $\imath$ th agent in the MAS is derived using fundamental laws of motion considering the front and rear axle wheels are lumped together as one. The states under study are lateral position, lateral velocity, yaw angle and yaw rate, with the control input being steering angle.δ. This developed model is used throughout the work for the control of MAS in order to achieve static consensus using state feedback approach and dynamic consensus using distributed MPC along with the study of platoon formation which is clearly stated in the next section.

Chapter 5

METHODOLOGY

5.1 OVERVIEW

In this chapter, the control mechanisms used in the analysis of vehicle platooning is characterized with their computational technologies. Control algorithm for state feedback controller to obtain static consensus in the agents along with its advantages and limitations in handling MAS is highlighted. DMPC control to obtain dynamic consensus with and without constraints are mentioned as different cases. Constrained control is studied under two cases using two different sets of initial conditions. The benchmark problem of platooning is analysed as an example with valid simulations under dynamic consensus control. Design of the controllers used in the work is condensed in section 5.2 which consists of two subsections covering the feedback control design and DMPC design. The latter subsections is again branched to give more details on optimization and constrained control using DMPC.

5.2 CONTROLLER DESIGN

In this study we aim to explore and analyse the techniques and methodologies for the controller design of state feedback and distributed MPC control to achieve consensus in MAS. We will investigate theoretical foundations, present practical algorithms, and discuss implementation considerations. Additionally, we will examine the case study of vehicle platoon formation using dynamic consensus algorithm. First section includes the study of state feedback approach where each agent measures its own state and uses this information to update its control inputs based on the observed states of other agents. By iteratively adjusting their control actions, the agents converge towards a shared state, thereby achieving consensus. Each agent $\imath$ employs a control law that incorporates the difference between its state and the consensus state. This difference represents the deviation of agent $\imath$ from the desired consensus state. The control law is designed to drive this deviation towards zero, promoting consensus. Most common approach is to design a linear state feedback controller, where the control input U_i for $\imath^{th}$ agent is computed as the product of feedback gain matrix K and the deviation in state. The feedback matrix is computed through pole placement technique.

The second section involves the DMPC control for dynamic consensus where different initial conditions are considered when taking the constrained control. In a distributed MPC framework, each agent possesses a model of the system dynamics and solves an optimization problem to determine its optimal control actions. The optimization problem involves minimizing a cost function that captures performance objectives while respecting constraints. The cost function is designed to achieve consensus. DMPC is an extension of Model Predictive Control (MPC) applied to a distributed setting. In traditional MPC, a centralized controller solves an optimization problem to find the optimal control inputs over a finite prediction horizon, considering the system model and constraints. DMPC, on the other hand, distributes the control problem among the agents in the system, and each agent optimizes its actions based on local information and interactions with neighboring agents. When considering dynamic consensus the local models and optimization problems of individual agents are interconnected through a communication network which can change over time. The agents exchange information regarding their states, control inputs, and optimization variables to collaboratively reach a consensus. This communication can occur directly between neighboring agents or through a network structure represented using graph and matrices. Examinations are done by considering constraints on the input as well as states under two different set of initial conditions, and also without constraints

5.2.1 State Feedback Control

A state feedback controller is a control system technique used in control theory to regulate or stabilize a dynamic system. It involves using the full state of the system, typically represented by a set of variables known as "state variables," to compute the control action. The state variables represent the internal state of the system and can include quantities such as position, velocity, acceleration, or any other relevant variables that describe the system's behavior. By measuring or estimating these state variables, the state feedback controller can determine the

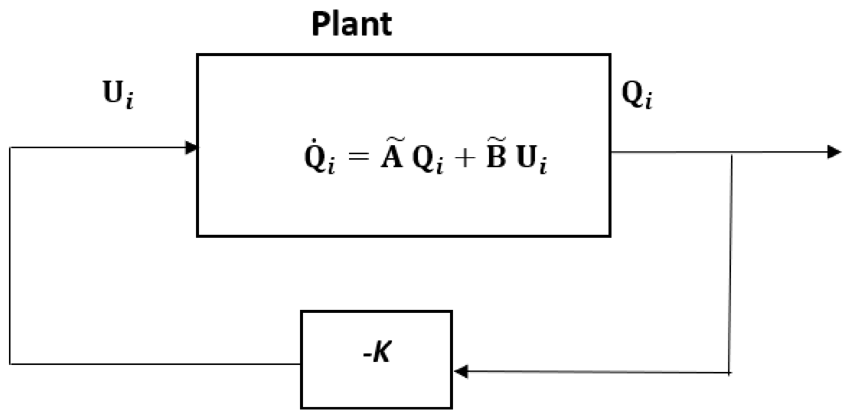


Figure 5.1: State Feedback Controller

appropriate control action to achieve the desired system behavior. The control action is typically computed using a linear combination of the state variables, multiplied by a set of control gains or coefficients. The control gains are chosen based on the system's dynamics and desired performance criteria. The control action is then applied to the system to influence its behavior. Thus the general objective of state feedback control is to utilize the full state information of a system to design control actions that stabilize the system, regulate its behavior and optimize performance. By leveraging state feedback, the control system can achieve desired stability, robustness, and performance characteristics for a wide range of dynamic systems. The state feedback control approach is implemented by computing the state feedback gain matrix. The control law for $\imath$ th vehicle is designed as:

$$U_i = -K\mathbb{L}Q_i \tag{5.1}$$

where U_i is the control vector and $\mathbb{K}$ is the state feedback gain matrix. The control law is set using the consensus protocol following the network topology as represented in Figure 5.1, so as the system under study attains static consensus as described in equation (3.5) based on this control strategy. Assumption is made such that the matrix pair ($\tilde{A}$,$\tilde{B}$) are controllable. The vehicle platoons is observed to settle on a common agreement as this feedback approach is implemented. In vehicle platooning, it is important to enforce constraints such as bounds on the control inputs or the velocity of the vehicles which is not possible in feedback control. Also feedback control only uses the current state information to determine the control inputs which is not desirable in the presence of disturbances. State feedback controller is limited to optimizing a single objective and cannot handle multiple objectives and trade-offs between them. All these limitations can be tackled by the extension of control strategy with an optimisation technique. Hence the control is modified using a distributed model predictive controller which can make the system under study to yield more optimised results. Also MPC focuses on the dynamic consensus of platoon configuration where the vehicles continue to be in the time varying trajectory which is more desirable.

5.2.2 Distributed Model Predictive Control

DMPC is an extension of Model Predictive Control (MPC) applied to a distributed setting. In traditional MPC, a centralized controller solves an optimization problem to find the optimal control inputs over a finite prediction horizon, considering the system model and constraints. DMPC, on the other hand, distributes the control problem among the agents in the system, and each agent optimizes its actions based on local information and interactions with neighboring agents. This section comprises of the detailed description regarding the conventional MPC and the distributive MPC its theoretical explanations, design procedures, the control algorithms and constraints handling taken into account for this specific study to attain dynamics consensus by taking the vehicle platoon case study.

Basic concepts of Model Predictive Controller

Model-based predictive control refers to a class of sophisticated control approaches that employ a process model to predict the behaviour of the controlled system in the future. It determines the control rule implicitly by solving a (possibly constrained) optimisation problem. This moves the focus for designing a controller towards modelling of the process to be controlled. Because such models are accessible in many sectors of engineering, also it eliminates the initial barrier to implementing control. Its implicit formulation preserves the physical understanding of the system characteristics, making controller tuning easier. MPC can even control systems that traditional feedback controllers cannot. The general objective of MPC is to optimize control actions over a finite time horizon, considering system dynamics, constraints, and performance objectives. It provides a systematic and flexible approach to achieve stability, reference tracking, and optimal control of dynamic systems in various applications. An overview of MPC and its key components are included here.

- Model: A mathematical model that represents the dynamics of the system being controlled is required. This model can be derived from first principles, obtained through system identification techniques, or developed using data-driven approaches.

- Prediction Horizon: MPC operates over a finite prediction horizon divided into discrete time steps. At each time step, the current state of the system is measured or estimated, and predictions of the system's future behavior are made over the prediction horizon.(P)

- Control Horizon: A finite control horizon, which is a subset of the prediction horizon is defined. The control horizon specifies the number of future time steps for which the control actions are computed.(M)

- Cost Function: A cost function is defined to quantify the performance objectives of the control problem. The cost function typically includes terms related to desired setpoints, system states, control effort, and constraints. The objective is to minimize this cost function over the prediction horizon.

- Optimization: It formulates an optimization problem based on the model, prediction horizon, control horizon, and cost function. The optimization problem seeks the control actions that minimize the cost function while satisfying system constraints.

- Constraint Handling: MPC can handle various constraints, such as state limits, control input limits, and other system-specific constraints. These constraints are incorporated into the optimization problem to ensure the control actions satisfy the constraints over the prediction horizon.

- Receding Horizon Control: MPC follows a receding horizon control approach. At each time step, the optimization problem is solved over the prediction horizon, but only the control actions for the first time step are implemented. After each control action is applied, the system evolves, and the process is repeated, shifting the prediction and control horizons.

- Feedback: Although MPC is a predictive control strategy, it typically incorporates feedback by continuously updating the prediction and control horizons with new measurements or state estimates. This allows the control actions to be adjusted in real-time based on the current system state.

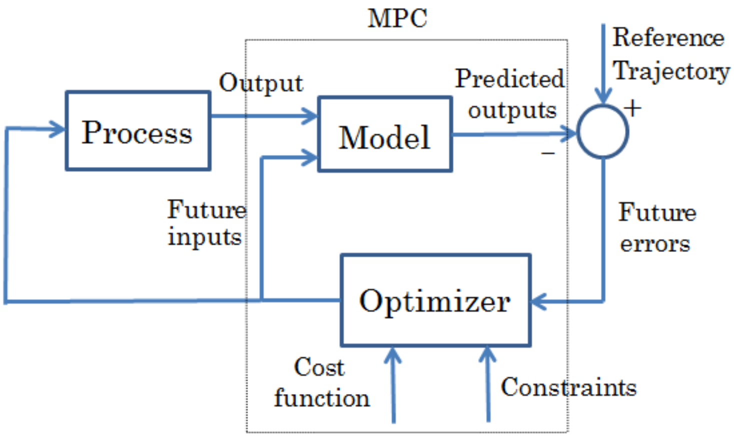


Figure 5.2: Block Diagram of MPC

- Implementation: It requires real-time computation to solve the optimization problem. Various numerical optimization techniques, such as quadratic programming or nonlinear programming, are used to efficiently solve the optimization problem within the control cycle.

The schematic representation of MPC is shown in Figure 5.2. To predict the present values of the output variables, a process model is employed. The Prediction block receives input from the residuals, which are the differences between the actual and predicted results. At each sample time, the predictions are employed in two types of MPC computations: set-point calculations and control calculations. Either form of calculation can incorporate inequality constraints on the input and output variables, such as upper and lower limits. Set points for the control calculations, also known as targets, are derived from an economic optimisation based on a steady-state model of the process, usually a linear model. Typical optimisation goals include maximisation of a profit function, minimization of a cost function, and maximisation of a production rate. The optimal values of set points fluctuate frequently as a result of changing process conditions, particularly variations in inequality constraints. Calculations are based on current measurements and predictions of the future values of the outputs. The objective of the MPC control calculations is to determine a sequence of control moves (that is, manipulated input changes) so that the predicted response moves to the set point in an optimal manner. The actual output y, predicted output $\hat{y}$ and manipulated input u for SISO control are shown in Figure 5.3. At the current sampling instant, denoted by k, the MPC strategy calculates a set of M values of the input u(k+ i- 1),i= 1, 2, . . . , M. The set consists of the current input u(k) and M- 1 future inputs. The input is held constant after the M control moves. The inputs are calculated so that a set

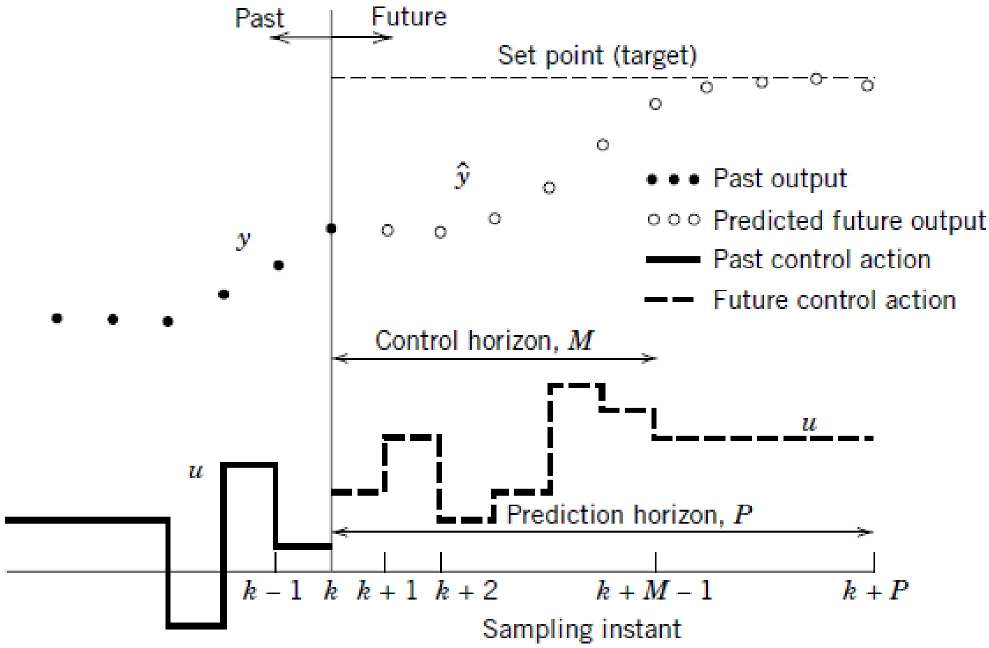


Figure 5.3: Receding Horizon Control

of P predicted outputs $\hat{y}$ (k + i), i = 1, 2, . . . , P. reaches the set point in an optimal manner. The receding horizon approach of MPC is one of its unique characteristics. At each sampling instant, a series of M control moves is computed, but only the first move is actually carried out. Once additional measurements are available, a new sequence is computed at the following sampling instant, this time implementing only the first input move. This makes a shift in the horizon. At each sample moment, this process is repeated. This receding horizon control makes MPC a feedback controller.

Distributed Model Predictive Controller Design

Dynamic Consensus and Distributed Model Predictive Control (DMPC) are two concepts that can be combined to achieve cooperative control and coordination in MASs. This concept is utilised here since one of the main objective behind this study is attaining dynamic consensus of vehicles in platoon. As the state feedback control law results in static consensus, the control law is modified using MPC algorithm to obtain dynamic consensus of vehicle system represented by equation (3.6). This section introduces some basic formulations and design of the controller to attain dynamic consensus of platoon system. For the system represented by the state and output equation as in (3.1) and (3.2). Assuming that the matrix pair ($\tilde{A}$,$\tilde{B}$) are controllable and the network is an undirected fixed topology as shown in Figure 5.4. The continuous state space

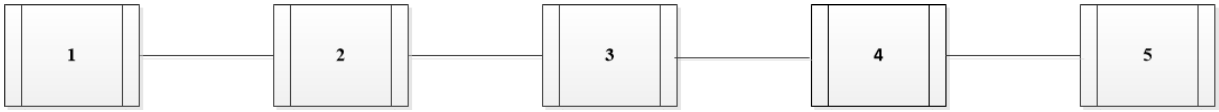


Figure 5.4: Communication topology of vehicles

model in equations (3.1) and (3.2) is discretized using zero order hold which is illustrated in Figure 5.4, resulting in the discrete system dynamics represented as:

$$Q_i(\dot{k}+1) = \tilde{A}_d Q_i\,(k) + \tilde{B}_d U_i\,(k)\,, i\varepsilon\vee \tag{5.2}$$

$$\tilde{Y} = \tilde{C}_d Q_i\,(k) \tag{5.3}$$

$\tilde{A}_d$ is the discrete system matrix, $\tilde{B}_d$ is the discrete input matrix and $\tilde{C}_d$ is the discrete output matrix. An augmented state space model is developed from the equation (5.2) with an incremental control $\Delta u_i(k) = u_i(k) - u_i(k-1)$ as the input. Let the incremental state vector be: $\Delta Q_i(k) = Q_i(k) - Q_i(k-1)$. The augmented state vector is given as: $Q_i(k+1) = [\Delta Q_i(k)^T \tilde{Y}_i(k)^T]^T$

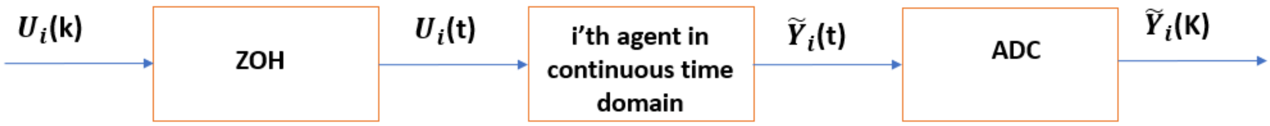


Figure 5.5: MAS discrete model

The augmented state space model is derived as:

$$Q_i(\dot{k}+1) = AQ_i\,(k) + BU_i\,(k) \tag{5.4}$$

$$\tilde{Y} = CQ_i\,(k) \tag{5.5}$$

where the sates $Q_i\ \varepsilon\ \Re^{n+p}$, the incremental control input $\Delta U_i\ \varepsilon\ \Re^{s}$, the system output $\tilde{Y}_i\ \varepsilon\ \Re^{p}$. A, B and C constitutes the augmented system matrix , input matrix and output matrix respectively.

The matrices are A= $\begin{bmatrix} A_d & 0' \\ C_d A_d & I_p \end{bmatrix}$, B= $\begin{bmatrix} A_d & 0' \\ C_d A_d & I_p \end{bmatrix}$ C= $\begin{bmatrix} 0'' & I_p \end{bmatrix}$ where 0' and 0" are null matrices of order $n \times p$ and $p \times n$.

The predicted control vector is given as:

$$U_i(k) = \left[u_i(k), u_i(k+1), ...u_i(k+w_c)\right]^T \tag{5.6}$$

, where w_c is the control horizon. Also the predicted state vector is defined as

$$Q_i(k) = \left[q_i(k), q_i(k+1), ...q_i(k+w_p)\right]^T \tag{5.7}$$

where w_p is the prediction horizon. The predicted output variables is computed from the output equation (3.2) and is expressed in compact form as

$$\tilde{Y}_i = \mathbb{G}Q_i(k) + \phi U_i(k) \tag{5.8}$$

where

$$\mathbb{G} = \begin{bmatrix} CA \\ CA^2 \\ . \\ . \\ . \\ CA^{w_p} \end{bmatrix} \tag{5.9}$$

$$\phi = \begin{bmatrix} CB & 0 & . & . & . & 0 \\ CAB & CB & . & . & . & 0 \\ CA^2B & CAB & . & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ CA^{w_p-1} & CA^{w_p-2} & . & . & . & CA^{w_p-wc} \end{bmatrix} \tag{5.10}$$

Optimization

For a given set point $\mathbb{C}_i(k)$ the objective of predictive control system is to bring the predicted output as close as possible to set point signal. Here the set point refers to the consensus point of the states of each agent. This objective creates a design to find the best control parameter U such that the difference between the consensus point and predicted output is minimized. The cost function of the vehicle system F_{veh} which reflects the objective function of the controller can be defined as

$$F_{veh} = \left[\mathbb{C}_i(k) - \tilde{Y}_i(k)\right]^T \left[\mathbb{C}_i(k) - \tilde{Y}_i(k)\right] + U_i(k)^T \mathbb{P} U_i(k) \tag{5.11}$$

where $\mathbb{P}$ is the weightage factor. The consensus point $\mathbb{C}_i(k)$ for $\imath$ th vehicle in each optimization window is computed to be the average consensus of the $\imath$ th vehicle and its neighbors. [45]. The first function in equation (5.11) specifies the objective of minimizing the deviation between the predicted output and consensus point signal. The second term reflects the consideration given to the size of the control vector U when the objective function is made to be small as possible. How large the control parameter U is, focus is given more to make the deviation in $\left[\mathbb{C}_i(k) - \tilde{Y}_i(k)\right]^T \left[\mathbb{C}_i(k) - \tilde{Y}_i(k)\right]$ as small as possible. To obtain the optimal control vector that will minimize the objective function we take the partial derivative of cost function and equate it to zero which is expressed as

$$\frac{\partial F_{veh}}{\partial U} = -2\phi^T C_{i(k)} - \mathbb{G}Q_{i(k)} + 2\left(\phi^T\phi + \mathbb{P}\right)U = 0 \tag{5.12}$$

This yields the optimised control vector expressed as

$$U = \left(\phi^T\phi + \mathbb{P}\right)^{-1} \phi^T \left(\mathbb{C}_i(k) - \mathbb{G}Q_i(k)\right) \tag{5.13}$$

Constrained Optimization in Distributed Model Predictive Controller

Constraints play a crucial role in DMPC as they ensure that the control actions remain within safe operating limits. Constraints can be of various types, such as input constraints, output constraints, state constraints, or even operational constraints specific to the system being controlled. Solving the constrained optimization problem in DMPC involves finding the optimal control inputs for each agent while ensuring that all the constraints are satisfied. The optimization problem can be formulated as a Quadratic Program (QP) or a Nonlinear Program (NLP), depending on the dynamics and constraints of the system. Agents iteratively solve their local optimization problems based on their local information and constraints. The solution to the local optimization provides the control inputs for the current time step. The system then proceeds to the next time step, and the optimization process is repeated in a receding horizon fashion, where the horizon shifts to the next time step. Simulation results in this work shows the degree of performance deterioration can be reduced if the constraints are incorporated in the implementation, leading to the idea of constrained control. Constrained optimisation technique performed here does not involve any other sets of computational methods for optimization but the combinations of the control approached of DMPC with the weighted average dynamic consensus algorithm. This work focuses on the implementation of constraints on the input as well as the states. The constrained control analysis is done as two sections where the initial computations involve imposing constraints on the states and inputs with the previous initial conditions used when the controlling is done without constraints. Later on analysis is done by imposing the same constraints by changing the initial conditions taken for the former analysis which is turns yields more efficient results. The performance of the control algorithm under these changing conditions are given in the simulation results on the next section.

5.3 SUMMARY

The control algorithms and its modifications applied on the state space model are analysed. Both the state feedback controller and the distributed MPC specifications and control flow to attain both static and dynamic consensus in order to find a solution to the benchmark problem of platooning is studied. Also the performance of the proposed system while handling constraints under different conditions are also examined in this section.

Chapter 6

SIMULATION RESULTS

6.1 OVERVIEW

The control algorithms analysed made on previous sections are validated through a MATLAB simulations in order to prove the attainment of static and dynamics consensus control of MAS. Platoon formation analysis is also done by considering five identical vehicles which are interconnected through a communication network and controlling is done by steering angle control. Section 6.2 gives the results on state feedback controller in achieving static consensus control of the five MAS. Simulations on DMPC with and without constraints is depicted on section 6.3 with further subdivisions to portray the effects when handling constraints under two different cases of initial conditions. These results also depicts how the vehicle platooning is made possible through consensus based control algorithm.

6.2 RESULTS ON STATE FEEDBACK CONTROLLER

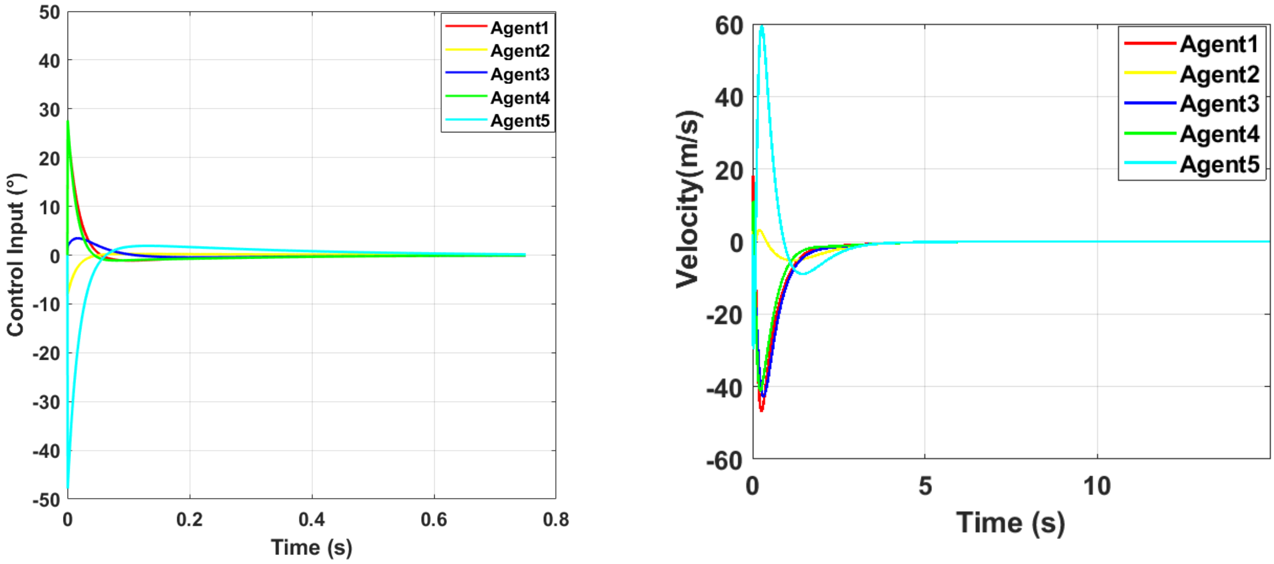
Static consensus control of five identical vehicles following the given undirected communication topology for the study of platooning problem is done by applying the consensus control algorithm given in equation (3.5) with the system dynamics in equation (4.7). This system is modified using the vehicle parameters mentioned in Table 6.1 and the state feedback control approach is applied. The values of state space matrices obtained by substituting the vehicle parameters on the state space model are: $\tilde{A} = \begin{bmatrix} 0 & 1 & 30 & 0 \\ 0 & -5.1 & 0 & -30.8 \\ 0 & 0 & 0 & 0 \\ 0 & -0.51 & 0 & -4.7 \end{bmatrix}$; $\tilde{B} = \begin{bmatrix} 0 & 90 & 0 & 64.8 \end{bmatrix}$.

The initial states of each agent used in the computation can be represented in order:

- The position of the five agents are y_i = $\begin{bmatrix} 50 & 25 & 85 & 67 & 20 \end{bmatrix}^T$ with unit as metre (m).
- The lateral velocity which is given as V_{LAi}= $\begin{bmatrix} 10 & 8 & 5 & 8 & 4 \end{bmatrix}^T$ for each agents respectively with unit metre per seconds(m/s).
- The yaw angle or vehicle's heading angle θ = $\begin{bmatrix} 4 & 8 & 4 & 5 & 12 \end{bmatrix}^T$ for each agents in order with unit in degrees (°).
- The yaw rate given as S= $\begin{bmatrix} 2 & 4 & 9 & 4 & 9 \end{bmatrix}^T$ for each of the five agents respectively with unit degree per seconds (°/s).

The state feedback control approach is done using the sate feedback gain matrix 'K' which is found using the pole placement technique. The K matrix obtained in this computation can be represented as: K= $\begin{bmatrix} 0.1520 & 0.0264 & 2.8340 & 0.1818 \end{bmatrix}$. All the four states of vehicle under study and the control input is depicted in the results. The given simulations demonstrate that the state feedback control technique aids the MAS in reaching static consensus.

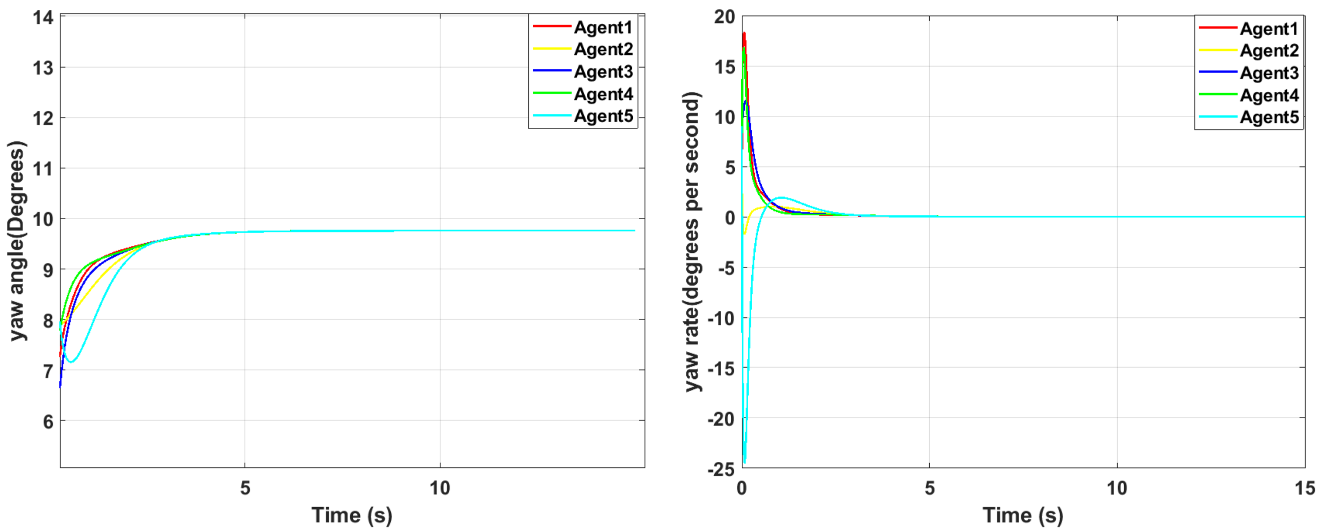
Figure 6.1 (a) depicts the control input to the five agents in MAS which is the steering angle whose value ranges from -50° to 30°. The velocity of the five identical vehicles reaching static consensus is depicted in Figure 6.1 (b). It can be observed that initially the velocities are at different ranges and it varies in both the directions making the states negative but as the time progresses it finally reaches the same velocity within five seconds and it remains in the same velocity indicating the static consensus. Figure 6.2 (a) depicts the yaw angle or the heading angle of the agent with respect to the reference co-ordinates. All the agents converges to the zero within five seconds and attain static consensus. Though the difference between the respective yaw angle of each vehicle is different at first later their difference is minimised to zero to attain the goal. The rate of change of heading angle of each agent converging to zero value and attaining static consensus is shown in Figure 6.2 (b) The convergence rate can be observed within five seconds and the value of yaw rate varies from -25 °/s to +20°/s for individual agents.



(a) Control input of the five agents

(b) Velocity of five agents

Figure 6.1: Static Consensus Control of MAS



(a) Yaw angle of five agents

(b) Yaw rate of five agents

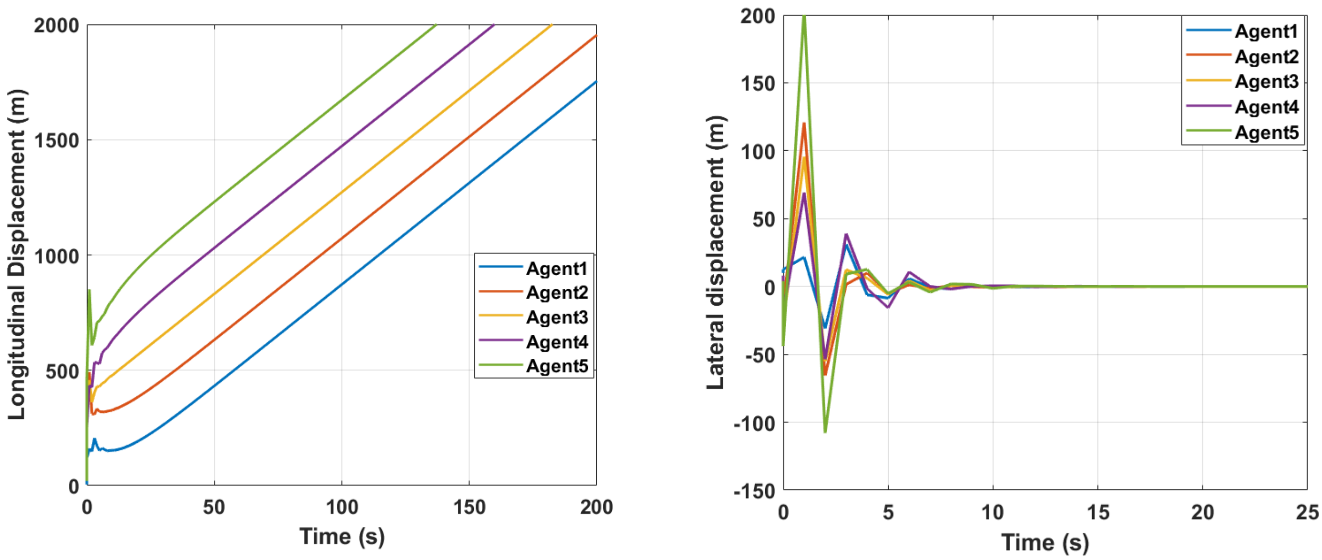
Figure 6.2: Static Consensus Control of MAS

6.3 RESULTS ON DMPC

The convergence of all the five ground vehicles to the desired states under platoon study is investigated using DMPC by implementing the consensus control rule provided in equation (3.6). The average consensus algorithm is applied on distributed computing technique in the system dynamics, which may be updated using the values received from the state space model. Simulations are done by imposing constraints on the states and control input.

6.3.1 DMPC Without Constraints

Convergence analysis of the vehicles on all the four states are initially done without implementing any constraints and the convergence rate and final consensus point is noted for further analysis on constrained optimization. The initial conditions used here for computations are same as that used in the state feedback control approach.

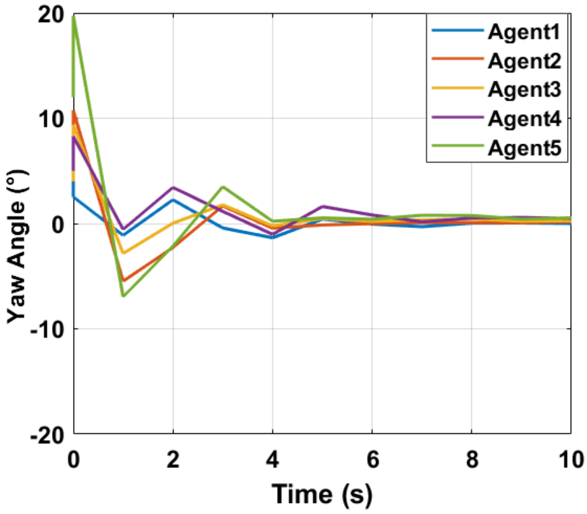


(a) Longitudinal position of five vehicles

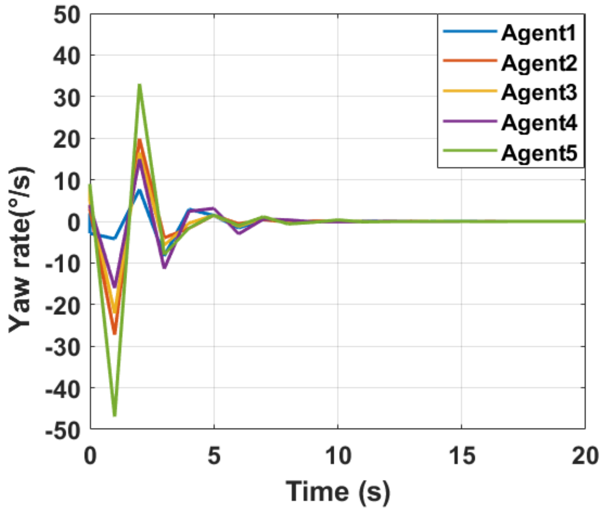
(b) Lateral position of five vehicles

Figure 6.3: Dynamic consensus of MAS without constraints

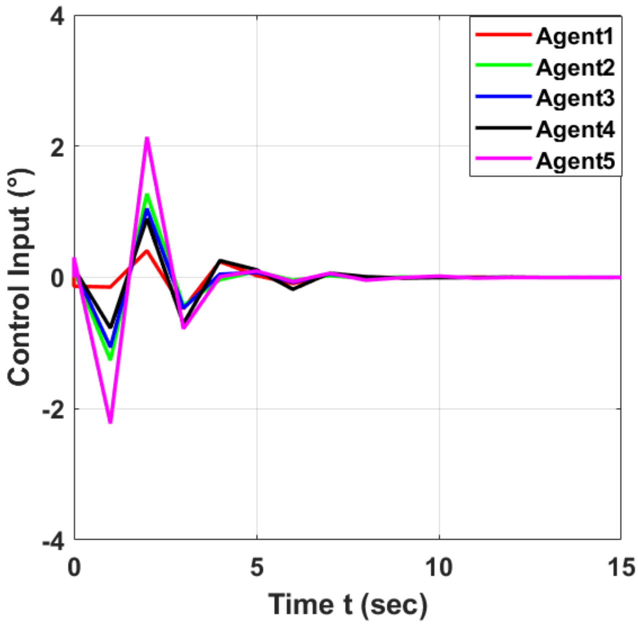
The longitudinal displacement of each vehicles is observed to attain consensus by maintaining an inter agent distance among them and continues to move in the time varying trajectory by maintaining the position between them. Figure 6.3 (a) depicts the longitudinal position of five vehicles in dynamic consensus without any constraints imposed on it.‘The position values shows a variance from 0 to 800 m for the five agents. Figure 6.3 (b) depicts the lateral posi-



(a) Yaw angle of five vehicles



(b) yaw rate of five vehicles



(c) Control input of five vehicles

Figure 6.4: Dynamic consensus of MAS without constraints

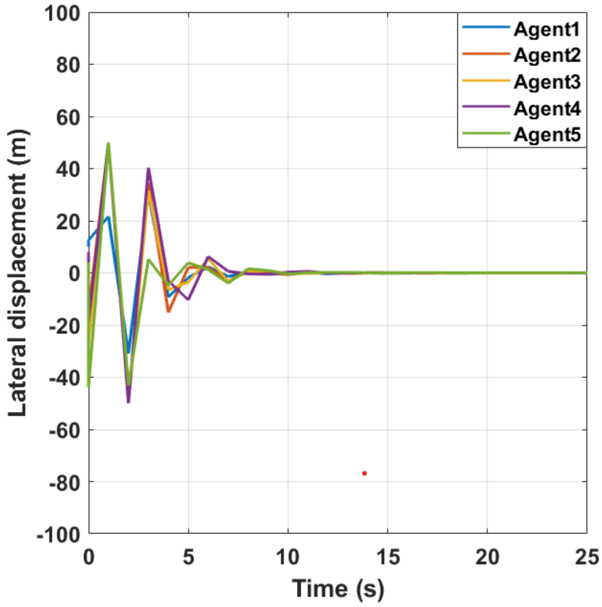
tion of five vehicles finally attaining dynamic consensus. Initially the vehicles to found to vary from the reference trajectory to both the directions ranging from -150 to 200 m then later on they attain a constant position with zero deviation within 10 seconds and continues to move through it. Figures 6.4 (a) and (b) illustrates the yaw angle and yaw rates of the vehicles attaining dynamic consensus respectively. The yaw angle shows deviations range from -10 to +20 degrees for each of the five vehicles and the differences in the values of each agent is found to be minimized within 10 seconds reaching dynamic consensus control. Value of yaw rate is found to vary from -50 to +50 degrees per seconds for the vehicles and within 10 seconds they are found in synchronisation. The control input of these system, which is the steering angle of the vehicle is depicted in Figure 6.4 (c) which is seen varying from -2.5 to +2.5 degrees and finally converging then continues in that value in the time varying conditions.

6.3.2 Constrained DMPC With First Set of Initial Conditions

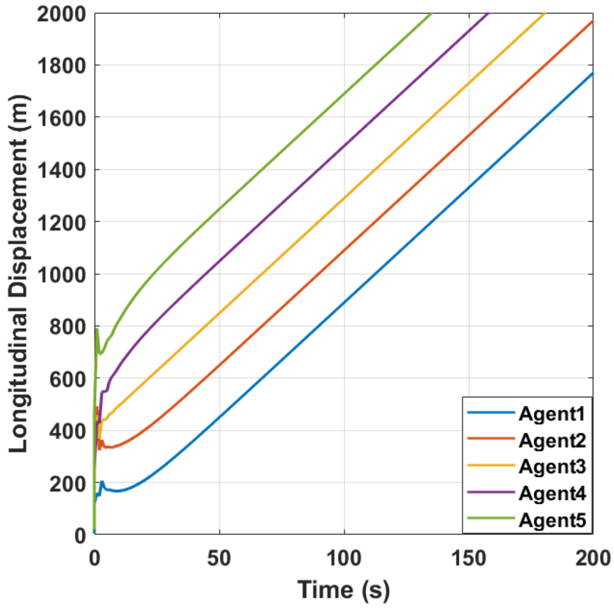
First step of DMPC design included the simulations done without any constraints. This section introduces some constraints onto the control input and states of the system. The control input constraint is given as $-1.5 \leq U_i \leq +1.5$. The lateral velocity limit is$-50 \leq V_{LA} \leq 50$. The yaw angle constraint ranges from $-15 \leq \theta \leq 15$. Yaw rate value should be within $-30 \leq S \leq 30$. The initial conditions used here is same as that used in the previous controller section which are:

- y_i = $\begin{bmatrix}50 & 25 & 85 & 67 & 20\end{bmatrix}^T$ m
- V_{LAi}= $\begin{bmatrix}10 & 8 & 5 & 8 & 4\end{bmatrix}^T$ m/s
- θ = $\begin{bmatrix}4 & 8 & 4 & 5 & 12\end{bmatrix}^T$ (°)
- S= $\begin{bmatrix}2 & 4 & 9 & 4 & 9\end{bmatrix}^T$ (°/s)

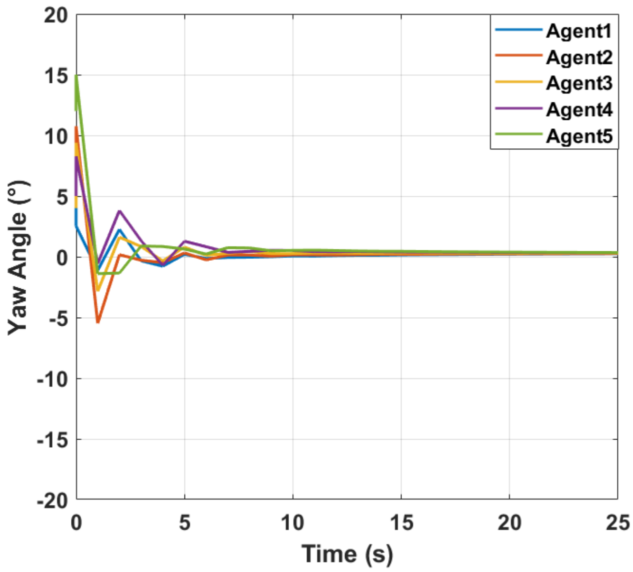
The simulations prove that the performance of the system is enhanced and the states and control input are within the range of constraints imposed. Figure 6.5 (a) depicts the lateral displacement of five vehicles approaching dynamic consensus and the position is maintained within the applied values ranging from -0 to +50 m. Figure 6.5 (b) depicts the longitudinal position of vehicles moving together by maintaining an inter vehicular distance and continues in that trajectory.



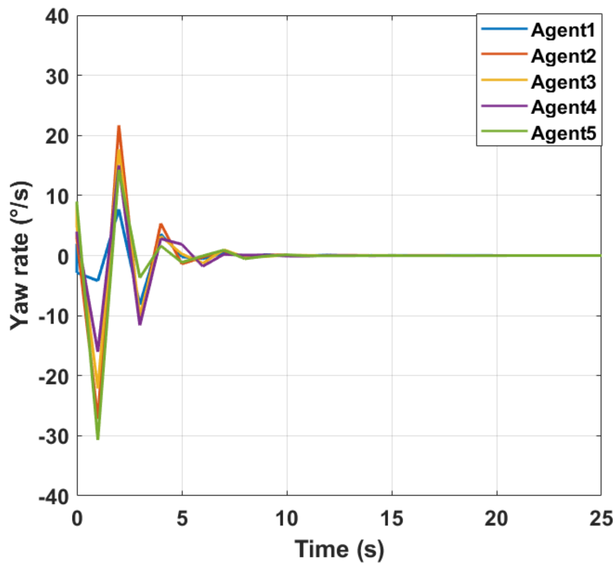
(a) Lateral displacement of five vehicles



(b) Longitudinal displacement of five vehicles



(c) yaw angle of five vehicles



(d) Yaw rate of five vehicles

Figure 6.5: Dynamic consensus of MAS with constraints with first set of initial conditions

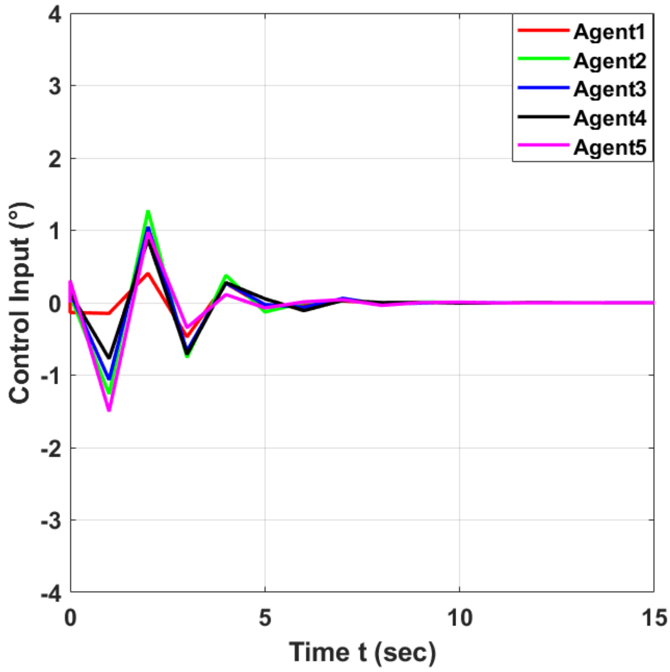


Figure 6.6: Dynamic consensus of MAS with constraints with first set of initial conditions

Figure 6.5 (c) and (d) portrays the yaw angle and yaw rate of the five vehicles in dynamic consensus. The changes on imposing constraints are clearly depicted in this figure as in previous case where there was no constraints the values had a grater deviation but here the difference is much smaller. The yaw angle θ ranges from -5 to +15 which is within the limits and the yaw rate S varies from -30 to +25 which is also within the limits. Control input to the five vehicles in the platoon system is given in Figure 6.6 whose value ranges from -1.5 to +1.5 degrees which is followed within the given limit. The value of U_i in the previous case was in the range of 2.5 degrees. The changes in the limits shows the constraint handling performance of DMPC.

6.3.3 Constrained DMPC With Second Set of Initial Conditions

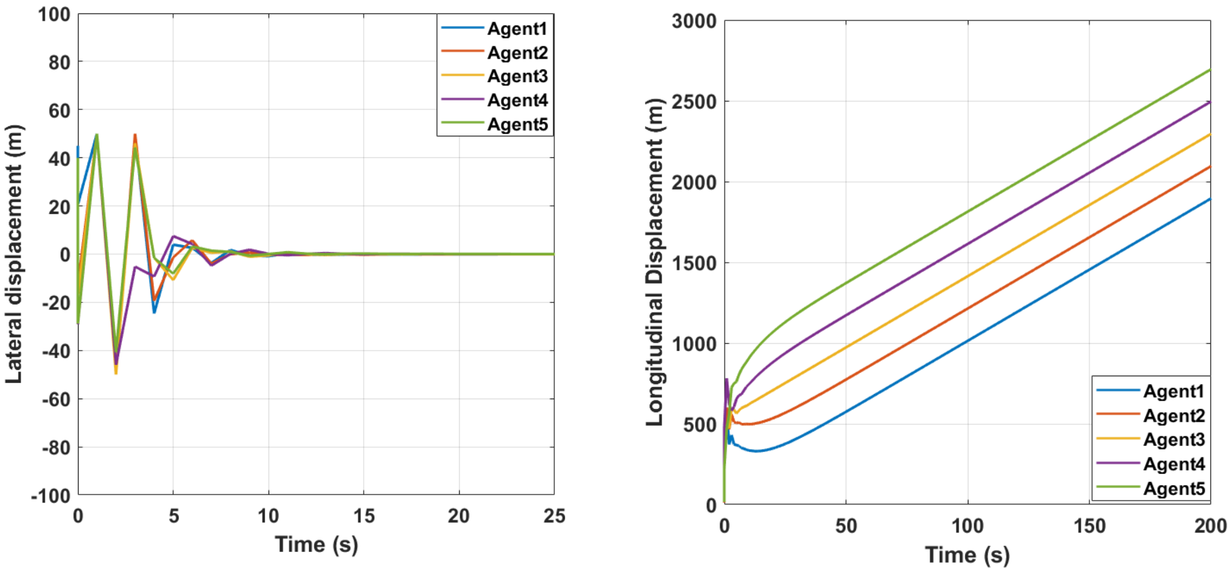
This section also handles constraints which is same as the previous section but the initial states of each vehicles is changed to different sets and the performance of each state attaining dynamic consensus is analysed through simulations. The second set of initial conditions used for the computations is given as:

- The position of the five vehicles are $y_i = \begin{bmatrix} 15 & 12.5 & 45 & 32 & 16 \end{bmatrix}^T$ with unit as metre (m).

- The lateral velocity is given as V_{LAi}= $\begin{bmatrix}45 & 28 & 35 & 6 & 40\end{bmatrix}^T$ for each agents respectively with unit metre per seconds(m/s).

- The yaw angle or vehicle's heading angle θ = $\begin{bmatrix}14 & 10 & 6 & 12 & 5\end{bmatrix}^T$ for each agents in order with unit in degrees (°).

- The yaw rate given as S= $\begin{bmatrix}1.2 & 7 & 10 & 14 & 6\end{bmatrix}^T$ for each of the five vehicles respectively with unit degree per seconds (°/s).



(a) Lateral displacement of five vehicles

(b) Longitudinal displacement of five vehicles

Figure 6.7: Dynamic consensus of MAS with constraints with second set of initial conditions

The constraints on the states and control input used in this part of simulations is similar to that in case of first set of initial conditions, which are:

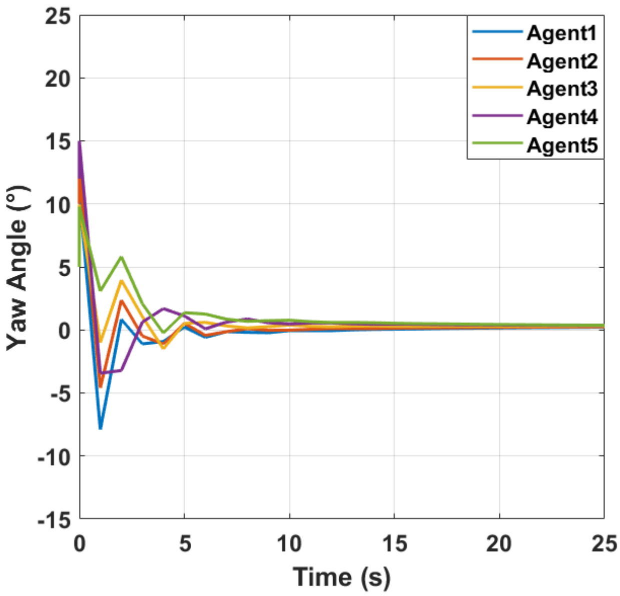
- $-1.5 \leq U_i \leq +1.5$

- $-50 \leq V_{LA} \leq 50$

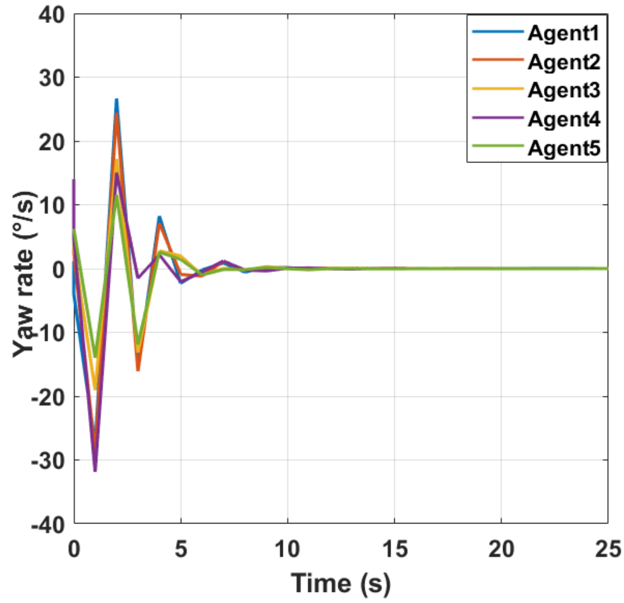
- T$-15 \leq \theta \leq 15$

- $-30 \leq S \leq 30$

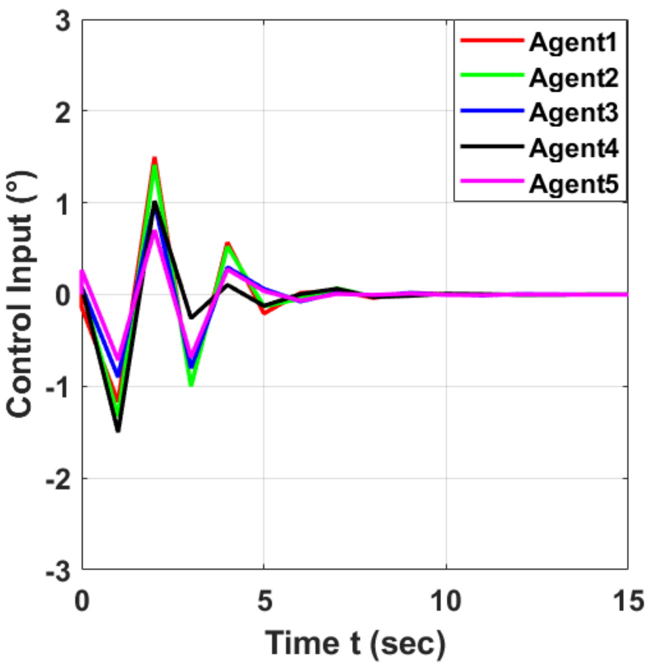
Lateral and longitudinal position on observation are given in Figures 6.7 (a) and (b), yaw angle and yaw rate are shown in Figure 6.8 (a) and (b). Simulation results depicted on these



(a) Yaw angle of five vehicles



(b) Yaw rate of five vehicles



(c) Control input of five vehicles

Figure 6.8: Dynamic consensus of MAS with constraints with second set of initial conditions

figures proves that the five vehicle continues to be in dynamic consensus and maintains the state values within the constraints imposed. Figure 6.8 (c) depicts the control inputs on the MAS which is also found to be under the constrained control approach of DMPC.

6.4 SUMMARY

The simulation results obtained by applying state feedback controller is analysed on the first part of the section which shows that MAS attain static consensus. The next part of this section includes the results of distributed model predictive controller which focuses on the dynamic consensus of vehicle platoons. DMPC control without constraints is done in the first part of vehicle platoon study. Then constrained control of DMPC is done using two different values of initial states. All the four states: Lateral position, lateral velocity , yaw angle and yaw rate is observed on dynamic consensus control under varying conditions and verified the study of platoon formation.

Chapter 7

CONCLUSION

The main aim of the project work is to attain both static and dynamic consensus control of linear distributed MAS. The former one is studied using state feedback controller and the latter one using distributed MPC control strategy. Also the benchmark problem of vehicle platoon formation is examined using the dynamic consensus algorithm and the results are validated through simulations. As part of phase one of the project work the problem identification is done and conducted literature studies from various reference works like conferences, books and other sources mentioned below, in relation within the area of the MAS and identified a suitable MAS. Here homogeneous linear ground vehicle is considered as agents for MAS design. Its modelling is done by representing it as an equivalent 3 DOF dynamic bicycle model with linear tires and the state space model is generated. Four states under study, derived from the model includes the position, velocity, yaw angle and yaw rate.

Further works on this project includes the controller design and simulations. Two controllers are used for the research work whose design is based on the, average consensus algorithm. In feedback controller design the state feedback gain matrix K is computed using pole placement technique and the static consensus algorithm is incorporated in it. Verification of results in this controller proves that the agents achieve static consensus. DMPC design involves the combination of the concepts of both the receding control approach of MPC in a distributed manner along with the dynamic consensus algorithm. The specific property of MPC optimization in constraint handling is taken into examinations in this research by analysing the control strategy without constraints initially. Later constraints are imposed on states and control inputs and is incorporated into the control algorithm with computations done as two stages of initial states. Simulation results on these control methodologies depicts that all the states in the model attains dynamic consensus and also prove to be within the limit of constraints thereby finding a solution to the benchmark problem of platooning. The use of distributed MPC helps in reducing the complexity as well as computational pressure in the system. Further investigations related to this work includes the use of heterogeneous agents with other robust control strategies. Also by extending the optimization techniques to quadratic programming, fmincon and other programming techniques without increasing the complexity of the system.

REFERENCES

[1] R. Rajamani, *Vehicle dynamics and control*. Springer Science & Business Media, 2011.

[2] Y. Cao, W. Yu, W. Ren, and G. Chen, "An overview of recent progress in the study of distributed multi-agent coordination," *IEEE Transactions on Industrial informatics*, vol. 9, no. 1, pp. 427–438, 2012.

[3] Y. Li and C. Tan, "A survey of the consensus for multi-agent systems," *Systems Science & Control Engineering*, vol. 7, no. 1, pp. 468–482, 2019.

[4] A. Dorri, S. S. Kanhere, and R. Jurdak, "Multi-agent systems: A survey," *Ieee Access*, vol. 6, pp. 28 573–28 593, 2018.

[5] C. W. Reynolds, "Flocks, herds and schools: A distributed behavioral model," in *Proceedings of the 14th annual conference on Computer graphics and interactive techniques*, 1987, pp. 25–34.

[6] T. Vicsek, A. Czirók, E. Ben-Jacob, I. Cohen, and O. Shochet, *Physical review letters*, vol. 75, no. 6, p. 1226, 1995.

[7] A. Jadbabaie, J. Lin, and A. S. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Transactions on automatic control*, vol. 48, no. 6, pp. 988–1001, 2003.

[8] Z. Liu and L. Guo, "Connectivity and synchronization of vicsek model," *Science in China Series F: Information Sciences*, vol. 51, no. 7, pp. 848–858, 2008.

[9] J. Tsitsiklis and M. Athans, "Convergence and asymptotic agreement in distributed decision problems," *IEEE Transactions on Automatic Control*, vol. 29, no. 1, pp. 42–50, 1984.

[10] F. Chen, W. Ren *et al.*, "On the control of multi-agent systems: A survey," *Foundations and Trends® in Systems and Control*, vol. 6, no. 4, pp. 339–499, 2019.

[11] Y. Chen, J. Lu, X. Yu, and D. J. Hill, "Multi-agent systems with dynamical topologies: Consensus and applications," *IEEE circuits and systems magazine*, vol. 13, no. 3, pp. 21–34, 2013.

[12] R. L. Berger, "A necessary and sufficient condition for reaching a consensus using degroot's method," *Journal of the American Statistical Association*, vol. 76, no. 374, pp. 415–418, 1981.

[13] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Transactions on automatic control*, vol. 49, no. 9, pp. 1520–1533, 2004.

[14] W. Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Transactions on automatic control*, vol. 50, no. 5, pp. 655–661, 2005.

[15] C. Tan, G.-R. Duan, and G. Liu, "Leader-following consensus problem of multi-agent networks based on homogeneous polynomial lyapunov functions," in *Proceedings of the 29th Chinese Control Conference*. IEEE, 2010, pp. 5747–5752.

[16] P.-P. Dai, C.-L. Liu, and F. Liu, "Consensus problem of heterogeneous multi-agent systems with time delay under fixed and switching topologies," *International Journal of Automation and Computing*, vol. 11, no. 3, pp. 340–346, 2014.

[17] D. Zhang and G. Duan, "Leader-following fixed-time output feedback consensus for second-order multi-agent systems with input saturation," *International journal of systems science*, vol. 49, no. 14, pp. 2873–2887, 2018.

[18] W. Zhu and D. Cheng, "Leader-following consensus of second-order agents with multiple time-varying delays," *Automatica*, vol. 46, no. 12, pp. 1994–1999, 2010.

[19] M. Lu and L. Liu, "Leader-following consensus of multiple uncertain euler–lagrange systems subject to communication delays and switching networks," *IEEE Transactions on Automatic Control*, vol. 63, no. 8, pp. 2604–2611, 2017.

[20] J. Yu and L. Wang, "Group consensus in multi-agent systems with switching topologies and communication delays," *Systems & Control Letters*, vol. 59, no. 6, pp. 340–348, 2010.

[21] S. Bittanti, A. Cenedese, S. Zampieri *et al.*, "Proceedings of the 18th ifac world congress," 2011.

[22] C. Tan, G.-P. Liu, and P. Shi, "Consensus of networked multi-agent systems with diverse time-varying communication delays," *Journal of the Franklin Institute*, vol. 352, no. 7, pp. 2934–2950, 2015.

[23] H. Zhao and J. H. Park, "Group consensus of discrete-time multi-agent systems with fixed and stochastic switching topologies," *Nonlinear Dynamics*, vol. 77, pp. 1297–1307, 2014.

[24] G. Wen, Y. Yu, Z. Peng, and H. Wang, "Dynamical group consensus of heterogenous multi-agent systems with input time delays," *Neurocomputing*, vol. 175, pp. 278–286, 2016.

[25] J. Qin, Q. Ma, Y. Shi, and L. Wang, "Recent advances in consensus of multi-agent systems: A brief survey," *IEEE Transactions on Industrial Electronics*, vol. 64, no. 6, pp. 4972–4983, 2016.

[26] L. Zou, Z.-D. Wang, and D.-H. Zhou, "Event-based control and filtering of networked systems: A survey," *International Journal of Automation and Computing*, vol. 14, no. 3, pp. 239–253, 2017.

[27] J. Hu, Z. Wang, F. E. Alsaadi, and T. Hayat, "Event-based filtering for time-varying nonlinear systems subject to multiple missing measurements with uncertain missing probabilities," *Information Fusion*, vol. 38, pp. 74–83, 2017.

[28] N. A. Oufroukh, A. Benine-Neto, Z. Yacine, S. Mammar, and S. Glaser, "Invariant set based vehicle handling improvement at tire saturation using fuzzy output feedback," in *2011 IEEE Intelligent Vehicles Symposium (IV)*. IEEE, 2011, pp. 1104–1109.

[29] C.-K. Chen and T.-K. Dao, "A study of bicycle dynamics via system identification," in *2010 International Symposium on Computer, Communication, Control and Automation (3CA)*, vol. 2. IEEE, 2010, pp. 204–207.

[30] H. Yoshida, S. Shinohara, and M. Nagai, "Lane change steering manoeuvre using model predictive control theory," *Vehicle System Dynamics*, vol. 46, no. S1, pp. 669–681, 2008.

[31] C. Zong, H. Liang, C. Tian, and R. Hu, "Vehicle chassis coordinated control strategy based on model predictive control method," in *The 2010 IEEE International Conference on Information and Automation*. IEEE, 2010, pp. 655–659.

[32] W. Yue, G. Guo, L. Wang, and W. Wang, "Nonlinear platoon control of arduino cars with range-limited sensors," *International Journal of Control*, vol. 88, no. 5, pp. 1037–1050, 2015.

[33] G. Liu, H. Ren, S. Chen, and W. Wang, "The 3-dof bicycle model with the simplified piecewise linear tire model," in *Proceedings 2013 international conference on mechatronic sciences, electric engineering and computer (MEC)*. IEEE, 2013, pp. 3530–3534.

[34] S. E. Shladover, "Longitudinal control of automotive vehicles in close-formation platoons," 1991.

[35] S. E. Shladover, C. Nowakowski, X.-Y. Lu, and R. Ferlis, "Cooperative adaptive cruise control: Definitions and operating concepts," *Transportation Research Record*, vol. 2489, no. 1, pp. 145–152, 2015.

[36] Y. Kebbati, N. Ait-Oufroukh, D. Ichalal, and V. Vigneron, "Lateral control for autonomous wheeled vehicles: A technical review," *Asian Journal of Control*, 2022.

[37] Y. Liu, C. Zong, and D. Zhang, "Lateral control system for vehicle platoon considering vehicle dynamic characteristics," *IET intelligent transport systems*, vol. 13, no. 9, pp. 1356–1364, 2019.

[38] Y.-C. Ho, "On centralized optimal control," *IEEE Transactions on Automatic Control*, vol. 50, no. 4, pp. 537–538, 2005.

[39] E. Camponogara, D. Jia, B. H. Krogh, and S. Talukdar, "Distributed model predictive control," *IEEE control systems magazine*, vol. 22, no. 1, pp. 44–52, 2002.

[40] M. Yan, W. Ma, L. Zuo, and P. Yang, "Distributed model predictive control for platooning of heterogeneous vehicles with multiple constraints and communication delays," *Journal of Advanced Transportation*, vol. 2020, pp. 1–16, 2020.

[41] L. Xiao and F. Gao, “Practical string stability of platoon of adaptive cruise control vehicles,” *IEEE Transactions on intelligent transportation systems*, vol. 12, no. 4, pp. 1184–1194, 2011.

[42] A. N. Venkat, “Distributed model predictive control: theory and applications,” Ph.D. dissertation, University of Wisconsin–Madison, 2006.

[43] R. R. Negenborn and J. M. Maestre, “Distributed model predictive control: An overview and roadmap of future research opportunities,” *IEEE Control Systems Magazine*, vol. 34, no. 4, pp. 87–97, 2014.

[44] R. Resmi, S. Mija, and J. Jacob, “Consensus control of two wheeled mobile robots with time delay,” in *2020 IEEE International Conference on Power Electronics, Drives and Energy Systems (PEDES)*. IEEE, 2020, pp. 1–6.

[45] ——, “Discrete laguerre-based model predictive control for dynamic consensus of a vehicle platoon with time delay,” *International Journal of Systems Science*, vol. 53, no. 12, pp. 2566–2583, 2022.

[46] S. Graffione, C. Bersani, R. Sacile, and E. Zero, “Model predictive control of a vehicle platoon,” in *2020 IEEE 15th International Conference of System of Systems Engineering (SoSE)*. IEEE, 2020, pp. 513–518.

[47] J. Zhan, Z.-P. Jiang, Y. Wang, and X. Li, “Distributed model predictive consensus with self-triggered mechanism in general linear multiagent systems,” *IEEE Transactions on Industrial Informatics*, vol. 15, no. 7, pp. 3987–3997, 2018.

[48] A. Khalifa, O. Kermorgant, S. Dominguez, and P. Martinet, “Platooning of car-like vehicles in urban environments: Longitudinal control considering actuator dynamics, time delays, and limited communication capabilities,” *IEEE Transactions on Control Systems Technology*, vol. 29, no. 6, pp. 2670–2677, 2020.

[49] S. E. Li, Y. Zheng, K. Li, and J. Wang, “An overview of vehicular platoon control under the four-component framework,” in *2015 IEEE Intelligent Vehicles Symposium (IV)*. IEEE, 2015, pp. 286–291.

[50] A. K. Bhoopalam, N. Agatz, and R. Zuidwijk, "Planning of truck platoons: A literature review and directions for future research," *Transportation research part B: methodological*, vol. 107, pp. 212–228, 2018.

[51] V. Lesch, M. Breitbach, M. Segata, C. Becker, S. Kounev, and C. Krupitzer, "An overview on approaches for coordination of platoons," *IEEE Transactions on Intelligent Transportation Systems*, vol. 23, no. 8, pp. 10 049–10 065, 2021.

[52] V. Milanés, S. E. Shladover, J. Spring, C. Nowakowski, H. Kawazoe, and M. Nakamura, "Cooperative adaptive cruise control in real traffic situations," *IEEE Transactions on intelligent transportation systems*, vol. 15, no. 1, pp. 296–305, 2013.

[53] H. Liu, X. Kan, S. E. Shladover, X.-Y. Lu, and R. E. Ferlis, "Impact of cooperative adaptive cruise control on multilane freeway merge capacity," *Journal of Intelligent Transportation Systems*, vol. 22, no. 3, pp. 263–275, 2018.

[54] E. H. Lim and J. K. Hedrick, "Lateral and longitudinal vehicle control coupling for automated vehicle operation," in *Proceedings of the 1999 American Control Conference (Cat. No. 99CH36251)*, vol. 5. IEEE, 1999, pp. 3676–3680.

[55] A. Bayuwindra, J. Ploeg, E. Lefeber, and H. Nijmeijer, "Combined longitudinal and lateral control of car-like vehicle platooning with extended look-ahead," *IEEE Transactions on Control Systems Technology*, vol. 28, no. 3, pp. 790–803, 2019.