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Strain Energy based Modeling of Soil liquefaction Using Data Driven Techniques

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ABSTRACT
This paper presents the application of Gaussian Process Regression (GPR) and M5 Model Tree as two alternative data driven modeling practices for prediction of soil liquefaction. The initial effective mean confining pressure (σ'_{mean}), initial relative density after consolidation (D_r), percentage of fines content (FC); uniformity coefficient (C_u); Coefficient of curvature (C_c), mean grain size (D_{50}) etc. are used as model inputs to predict strain energy density (W) required for triggering the liquefaction. The performance evaluation criteria like Mean Absolute Relative Error (MARE), Coefficient of Correlation (R), Root Mean Square Error (RMSE) for the validation datasets are found to be 6.381, 0.849 0.266 respectively. Use of multiple statistical criteria and graphical plots confirmed the superiority of PuK Kernel based GPR model over five different empirical models, two Linear Genetic Programming (LGP) based expressions, Artificial Neural Network (ANN) and M5Model tree based predictions. Further, a parametric sensitivity analysis performed on input parameters showed that σ'_{mean} is the most influencing predictor to explain the variations of the capacity energy than other input parameters.
Keywords: Liquefaction, Strain Energy, Kernel, GPR, Data Driven Techniques

1. INTRODUCTION

Soil liquefaction is one of the most complex phenomena studied by the geotechnical investigators. Liquefaction usually occurs when the pore water pressure increases to carry the overburden stress and Darve (1996) considered liquefaction as a specific feature of loose and saturated sandy soils. The available procedures for evaluation of liquefaction can be categorized broadly as stress-based procedures (Seed and Idriss, 1971), strain-based procedures (Dobry et al. 1982) and energy-based procedures (Figueroa et al., 1994; Liang 1995; Ostadan et al., 1996; Green 2001).The stress-based procedure, perhaps the most widely-used liquefaction assessment method, is mainly empirical and based on laboratory and field observations. The shear stress level and number of cycles are the main criteria in the stress-based procedure. The strain based method was derived from the mechanics of two interacting idealized sand grains and then generalized for natural soil deposits and it is based on the hypothesis that pore pressure initiates to develop when shear strain surpasses a threshold shear strain. The uncertainty related to random loading persists in both of these methods. The basic principles of both the stress and strain methods are incorporated in the formulation of the energy-based method. In this method, the amount of total strain energy at the triggering of liquefaction is obtained from laboratory testing or field recorded data. The instantaneous energy and its summation overtime intervals are computed until the onset of liquefaction, which is used as the measures of the capacity of the soil sample against initial liquefaction occurrence in terms of the strain energy (capacity energy).Over the years the researchers presented many analytical models for prediction of liquefaction triggering and based on experimental observations. These models include simple linear relations and the evolution of soft computing methods was a major breakthrough in this field. Artificial Neural Networks (ANN), perhaps the most popular data driven technique used for the evaluation of the liquefaction potential (Goh, 1994; 2002) and many of them used stress or strain based database for the evaluation. Baziar and Jafarian (2007) developed an ANN model for evaluation of the liquefaction potential based on the energy concepts. Chen et al. (2005) presented seismic wave energy-based method with back-propagation neural networks to assess the liquefaction probability. But the ANNs are not usually capable of providing practical prediction equation, involves complex process of parameter setting and the structure (architecture) is to be identified *apriori*. Baziar et al. (2011) utilized an evolutionary approach based on Genetic Programming (GP) for estimation of capacity energy of liquefiable soils. Some of the researchers used different variants of GP for prediction of soil liquefaction including linear GP and multi-expression programming (MEP) for evaluation of soil liquefaction (Alavi et al., 2011; Alavi and Gandomi 2012). This paper presents the application of Gaussian Process Regression (GPR) and M5 Model Tree as two alternative data driven modeling practices for prediction of soil liquefaction

In this present study, the usefulness of two recently developed data driven techniques namely Model Tree and Gaussian Process Regression (GPR) is investigated to obtain generalized relationships between the energy per unit volume dissipated during liquefaction and the soil initial parameters. This level of imparted energy density, denoted as capacity energy of the soil, indicates whether liquefaction is triggered in the soil deposit. Further, the prediction performance of the derived correlations was compared with that of different models already available.

2. MATERIALS AND METHODS

In this section the basic principles of two algorithms-M5 model trees and GPR used in this study are presented

2.1 M5 Model Tree

M5 Model Tree (MT) is a popular machine learning technique proposed by Quinlan (1992) used for solving regression problems through classification and decision making. It follows a modular approach so that the entire domain is divided into sub-domains and separate multi-linear regression models are developed for each of them. Therefore it formulates many piecewise linear models to approximate the non-linear relationship between the input variables and output variable. In the first stage, a decision tree is created following a splitting criterion. The one which uses standard deviation reduction (SDR) as the splitting criteria is called as M5 learning algorithm (Witten and Frank 2005). In this method the standard deviation of the class values that reach a node is treated as a measure of the error at that node and the expected reduction in this error as a result of testing each attribute at that node is calculated. The computation of SDR can be represented as follows:

$$\sigma_R = \sigma(N) - \sum \frac{|T_i|}{|N|} \sigma(T_i) \quad [1]$$

where, σ_R= standard deviation reduction; N is the total number of training samples; T_i is the training samples of i^{th} sub-domain; $\sigma(N)$ and $\sigma(T_i)$ are the standard deviations of complete training samples, and i^{th} sub-domain samples respectively. The computational process and decision making is depicted in Figure 1. Figure 1 represents the modular division of domains to sub-domains followed by model selection (tree building) corresponding to a 2-D input domain of parameters x_1 and x_2.

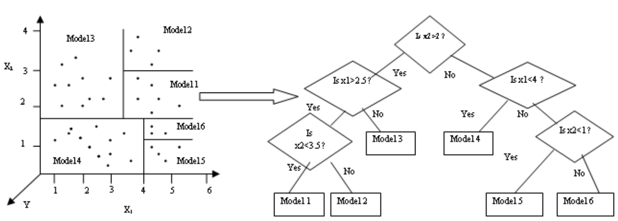

Figure 1. Data division and example of Model Tree (from Solomatine and Xue (2004))

The splitting continues till the class values of all the instances that reach a node varies negligibly or only a few instances remain. Then the model improvement is done by two ways : by performing 'pruning' and 'smoothing' operations, which reduce the effect of 'over fitting' and sharp discontinuities between different sub-classes, which happen especially when the dataset is very small (Jothiprakash and Kote 2011). A more detailed description of the theory behind model trees and pictorial representations can be found elsewhere (Witten and Frank 2005). The use of M5 model tree does not involve the setting of any algorithm specific user defined parameters (Singh et al., 2010) and another major advantage of M5 Model Tree is that, it combines several simple linear relations and hence more transparent and acceptable by decision makers.

2.2 Gaussian Process Regression

GP regression provides probabilistic models work on the assumption that adjacent observations should convey information about each other (Williams and Rasmussen, 1996; Rasmussen and Williams 2006). This is a natural generalization of the Gaussian distribution, whose mean and covariance are a vector and matrix, respectively. A GP is defined as a collection of random variables, any finite number of which has a joint multivariate Gaussian distribution. Let $\chi \times \gamma$ represent the domains of inputs and outputs, respectively, from which n pairs (x_i, y_i) are drawn independently and identically distributed. For regression, assume that $y \subseteq \Re$; then, a GP on χ is defined by a mean function $\mu: \chi \rightarrow \Re$ and a covariance function $k: \chi \times \chi \rightarrow \Re$. The main assumption of GP regression is that y is given by $y = f(x) + \xi$, where $\xi \sim \mathrm{N}(0, \sigma^2)$. The symbol ~ in statistics means sampling for. In GP regression, for every input x there is an associated random variable f(x) drawn from the Gaussian process on χ specified by k. i.e., $Y = (y_1, \ldots\ldots y_n) \sim \mathrm{N}(0, K + \sigma^2 \mathrm{I})$, where $K_{ij} = k(x_i, x_j)$, and I is the identity matrix. As $Y/X \sim \mathrm{N}(0, K + \sigma^2 \mathrm{I})$ is normal, so is the conditional distribution of test labels given the training and test data of $p(Y_*/Y, X, X_*)$. Then, one has $Y_*/Y, X, X_* \sim \mathrm{N}(\mu, \Sigma)$, where

$$\mu = K(X_*, X)\left(K(X, X) + \sigma^2 \mathbf{I}\right)^{-1} Y \quad [2]$$

$$\Sigma = K(X_*, X_*) - \sigma^2 \mathbf{I} - K(X_*, X)\left(K(X, X) + \sigma^2 \mathbf{I}\right)^{-1} K(X, X_*) \quad [3]$$

If there are n training data and n*test data, then $K(X, X_*)$ represents the n×n*matrix of covariance evaluated at all pairs of training and test data sets and similarly for the other values $K(X, X), K(X_*, X), K(X_*, X_*)$ where X and Y is the vector of training data and training data labels yi, whereas X*is the vector of test data. During the training process of GP regression models, one needs to choose a suitable covariance function as well as its parameters. In case of GP regression with a fixed value of Gaussian noise, GP model can be trained by applying Bayesian inference, that is maximizing marginal likelihood. This leads to the minimization of the negative log-posterior:

$$p(\sigma^2, k) = \frac{1}{2} y^T (K + \sigma^2 I)^{-1} y + \frac{1}{2} log|K + \sigma^2 I| - log\, p\,(\sigma^2) - log\, p\,(k) \quad [4]$$

To find the hyper parameters, the partial derivative of Equation (4) can be obtained with respect to σ^2 and k, and minimization can be achieved by gradient descent algorithm. A specified covariance function is required to generate a positive semi-definite covariance matrix *K,* where $K_{ij} = k(x_i, x_j)$. The Radial Basis function (RBF), which has a Gaussian base and the polynomial are the two popular Kernels used in GPR. The width of the Gaussian Kernel and the polynomial order (exponent) are the algorithm specific parameters in these popular Kernels (Pal and Deswal 2011). Üstün et al. (2006) proposed Pearson VII function universal kernel (PuK) and suggested it to be an alternative to the linear, polynomial and RBF kernels. The mathematical form of which is

$$\left(1\Big/\left[\left[1+\left(\frac{2\sqrt{|x_i-x_j|^2}\sqrt{2^{\left(\frac{1}{\omega}\right)}-1}}{\sigma}\right)^2\right]\right]^{\omega}\right) \qquad [5]$$

The Kernel specific parameter σ is the Pearson width, whereas ω is the tailing factor of the peak of the Kernel function. The technique

3. DATABASE

The energy required to liquefy a soil deposit is independent of the stress history and applied load pattern (harmonic or random (Liang et al., 1995). Therefore, there is a solid and simple linkage between laboratory and field behavior for energy based approach, in contrast with stress and strain-based approaches. Various studies have been carried out to propose energy based models relating pore water pressure increment ratio, r_u, to dissipated strain energy density, W, loading parameters such as cyclic stress ratio (CSR) or strain level, initial parameters of soils such as initial relative density (D_r), initial effective confining pressure (σ'_0), and some calibration parameters obtained from curve fitting of experimental data.

The evaluation is done by using the data (284) collected from previously published cyclic tests available in a published data (Baziar and Jafarian, 2007). The database contains 217 cyclic triaxial, 61 cyclic torsional shear and 6 cyclic simple shear tests on Monterey, Yatesville, Reid Bedford, on clean and silty sands. The two criteria that indicate the liquefaction triggering are: (1) initial liquefaction (r_u = 1) and (2) double amplitude of strain of 55% (ε_{DA} = 5%), whichever occurs first (Baziar and Jafarian, 2007). The database was divided into two separate groups denoted as training and testing sets consisting about 70% and 30% of data respectively. The training data was used for the learning process and the testing data was employed to measure the performance of the obtained model on data that play no role in building it. The data division is done in such a way that the maximum, minimum, mean, standard deviation and the coefficient of variation (CV) between the two data sets, is nearly same. Although normalization is not strictly necessary, better results are often reached after normalizing the variables. Further, normalization speeds up the learning process. These are mainly due to influence of unification of variables, no matter their range of variation (Alavi et al., 2010). Thus, the input and output variables were normalized between zero and 1. Selection of the optimal method for normalizing the data was on the basis of both controlling several normalization methods (Swingler, 1996) and the simplicity of the method.

Table 1. Statistics of Training and Testing Datasets

Property	σ'_{mean} (kPa)	D_r (%)	FC (%)	C_u	D_{50} (mm)	C_c	W (kJ/m^3)
	Training						
Max	294.0	105.1	100.0	5.880	0.46	1.610	34.9
Min	41.10	-44.5	0.000	1.570	0.03	0.740	0.3
SD	27.33	33.17	26.67	0.996	0.13	0.185	5.40
Mean	98.77	47.21	19.44	2.347	0.24	0.938	3.48
CV	0.276	0.70	1.371	0.424	0.55	0.197	0.001
	Testing						
Max	294.0	104.3	100.0	5.88	0.44	1.610	23.3
Min	41.10	-36.5	0.000	1.570	0.03	0.74	0.31
SD	28.85	34.75	25.57	1.080	0.12	0.203	4.14
Mean	97.44	47.70	22.76	2.434	0.21	0.949	3.05
CV	0.29	0.733	1.123	0.443	0.56	0.214	0.001

4. RESULTS AND DISUCSSION

First, the different empirical relations for the evaluation of liquefaction potential, reported in the literature are used. Different energy based models considered in the study are presented in Table 2.

Table 2 Empirical Models for evaluating liquefaction potential

$$Log(W) = 2.002 + 0.00477\sigma'_{mean} + 0.0116 D_r$$

Figueroa et al. (1994) (Model 1)

$$Log(W) = 2.062 + 0.0039\sigma'_{mean} + 0.0124 D_r$$

Liang et al. (1995) (Model 2)

$$Log(W) = 2.484 + 0.00471\sigma'_{mean} + 0.00052 D_r$$

Liang et al. (1995) (Model 3)

$$Log(W) = 1.164 + 0.01264\sigma'_{mean} + 0.02109 D_r$$

Dief and Figueroa (2001) (Model 4)

$$Log(W) = 2.4597 + 0.00448\sigma'_{mean} + 0.00115 D_r$$

Dief and Figueroa (2001) (Model 5)

$$Log(W) = 2.1028 + 0.004566\sigma'_{mean} + 0.005685 D_r + 0.001827 FC - 0.02868 C_u + 2.0214 D_{50}$$

Baziar and Jafarian (2007) (Model 6)

$$Log(W)_{LGP;I} = \frac{5}{4}(2\sigma'_{mean;n} D_{r;n} + D_{r;n} D_{50;n} + D_{r;n} D^2_{50;n} * (\sigma'_{mean;n} + D_{50;n} - (3\sigma'_{mean;n} - 6FC_n + 4C_{u;n})^2 - 1) + 2)$$

Alavi and Gandomi (2012) (Model 7)

$$Log(W)_{LGP;II} = \frac{5}{2} + 5\sigma'_{mean;n} D_{r;n} - 5(\sigma'_{mean;n} D_{r;n})^2$$

Alavi and Gandomi (2012) (Model 8)

In the Table 2, n stands for normalized and hence $\sigma'_{mean; n}$, $D_{r; n}$, FC_n, $C_{u; n}$ and $D_{50; n}$ respectively, denote the soil initial effective mean confining pressure, initial relative density after consolidation, percentage of fines content, coefficient of uniformity, and mean grain size in their normalized forms as given below:

$\sigma'_{mean; n} = \sigma'_{mean}/300$ [6]

$D_{r; n} = (D_r + 40)/150$ [7]

$FC_n = (FC + 40)/15$ [8]

$C_{u; n} = (1/6)*C_u$ [9]

$D_{50; n} = 2*D50$ [10]

Log(W) = 0.2*Log(W) [11]

Different model performance evaluation measures like correlation coefficient (R), coefficient of efficiency (E), root mean square error (RMSE), mean bias error (MBE), mean absolute relative error (MARE) are used for comparing the efficacy of different models used in this study. The performance evaluation of predictions by different energy based models stated in Table 2, are presented in Table 3.

Table 3. Performance evaluation of different energy based models. PEC-Performance evaluation criteria, M refers model, RM-Root mean square error; MB-Mean bias error; MA-Mean absolute relative error

PEC	M1	M2	M3	M4	M5	M6	M7	M8
Training set								
R	0.54	0.55	0.22	0.54	0.31	0.8	0.84	0.6
E	-0.14	-0.14	-0.41	-1.07	-0.42	0.6	-4.86	0.3
RM	0.48	0.48	0.53	0.64	0.54	0.3	1.08	0.38
MB	-0.25	-0.24	-0.31	0.09	-0.32	-0.003	-1.03	-0.05
MA	1.02	1.02	11.6	22.8	20.4	12.1	32.5	8.6
Testing set								
R	0.59	0.58	0.40	0.61	0.47	0.8	0.82	0.6
E	0.04	0.01	-0.19	-1.03	-0.19	0.6	-5.04	0.4
RM	0.44	0.45	0.49	0.64	0.49	0.7	1.11	0.3
MB	-0.21	-0.19	-0.26	0.15	-0.28	0.002	-1.05	-0.01
MA	11.45	11.82	10.69	16.49	10.64	6.5	33.41	8.7

For analyzing the result, log of the values of the strain energy density, obtained by applying the empirical equations were used. Analyzing the coefficient of correlation we can see the performance given by various empirical equations. The first five equations used (Figueroa et al.,1994, Liang et al., 1995; Dief and Figuera, 2001), were derived by performing a multiple linear regression (MLR) analysis, and developed a relation taking into account only two of the initial soil parameters (σ'_{mean}, D_r). Baziar and Jaffarian (2007) used an ANN model, and developed a relation taking into account five of the initial soil parameters(σ'_{mean}, D_r, FC, C_u and D_{50}). LGP model by Alavi and Gandomi (2012) developed 2 relations taking into account five of the initial soil parameters (LGP(I)) (σ'_{mean}, D_r, FC, C_u and D_{50}), and two of the soil parameters((LGP (II))(σ'_{mean}, D_r), but in their normalized form.

It was seen from the output analysis that, with the exception of the training data, the equation created by LGP(II) produced a better result than those generated by MEP. Also LGP(I) model developed a better result than that developed by Baziar and Jaffarian (2007). The result demonstrated that the LGP and MEP based formulas with five inputs significantly outperformed those using two inputs.

From the above tables it is clear that the several empirical or LGP model equations have different results and this will introduce considerable uncertainties in liquefaction assessment. ANN and LGP based model involves complex step of control parameter setting and the complexity enhances as there are many such control parameters in this methods. So it is logical to investigate in this direction and solicit experiments using alternative predictive methods. In this context, the M5 model tree and a Kernel based evaluator-the Gaussian Process Regression (GPR) are chosen as two candidate methods. In these first the M5 model tree algorithm is considered, in which no step of control parameter setting except the steps of pruning and smoothing.

The GPR modeling is experimented with different Kernals linear, PuK and the Gaussian. Several trials were done for each type of kernel varying the Kernel specific parameters. The noise was kept fixed for each trial and the trial which gave maximum correlation coefficient for both training and testing data sets and minimum error was selected for each type of kernel The results of performance evaluation of different data driven models used in this study are summarized in Table 4. The performance evaluation of two best models M5 model tree and GPR Gaussian Kernel are presented in Figure 2. From Table 4 it is clear that Model Tree and GPR have shown a fairly good predictions and a comparison with the statistics given in Table 3 confirms the superiority of these alternative methods. The graph plotted with 10% error line, shows that majority of the points lie between that error lines So it can be concluded that predictions made with these tools are reliable.

Table 4. Performance evaluation of data driven models

	Model Tree		GPR - PuK	
PEC	Training	Testing	Training	Testing
R	0.789	0.745	0.887	0.849
E	0.632	0.619	0.687	0.651
RMSE	0.222	0.278	0.119	0.266
MARE	13.11	6.904	12.89	6.381
MBE	0.434	0.119	0.212	0.084

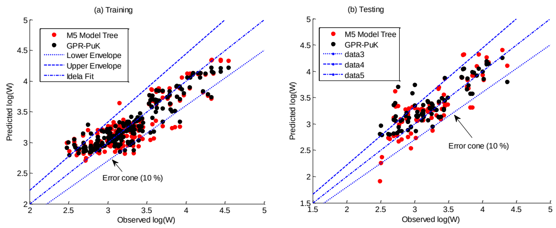

Figure 2. Scatter plots of predictions of Training and testing datasets

The results showed that the GPR based method performs better in terms of fitting statistics and error measures with less bias, when compared with other empirical expressions. The performance is close or equally good with ANN model by Baziar and Jafarian (2007). The Kernel based estimates in GPR involves very less number of control parameters to optimize, when compared with ANN or LGP methods, which is an added advantage of the present approach.

In the next, the model sensitivity (S, in %) is done to find the influence of different input parameters. In this process, the sensitivity is computed upon the best model (GPR-PuK). It is found out by changing the variables one by one by 30 % and determining the sensitivity as follows (Samui 2012):

$$S = \frac{1}{N_t}\sum_{j=1}^{j=N_t}\left(\frac{\%Change\ in\ output}{\%Change\ in\ input}\right)_j * 100 \qquad [12]$$

where N_t is the number of testing datasets.

The results of sensitivity analysis are presented in Figure 3. From the plot, it is clear that the confining pressure is the most influencing parameter to trigger soil liquefaction. The effect of fine content, uniformity coefficient and coefficient of curvature are similar.

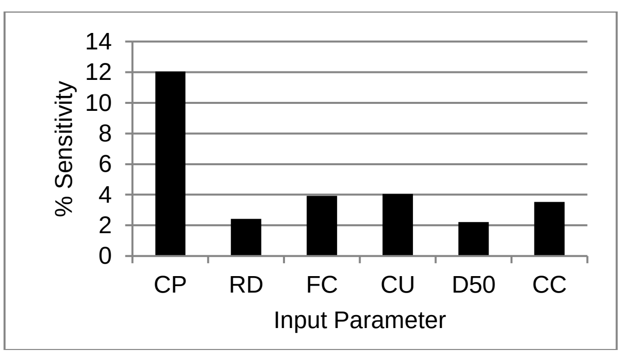

Figure 3. Sensitivity of Input parameters (CP- Confining pressure; RD-relative density; FC-Fine Content; CU-Uniformity Coefficient; D50 – 50 % finer CC-Coefficient of curvature Pressure)

5.CONCLUSION

This study proposed the application of M5 Model Tree and Gaussian Process Regression (GPR) for prediction of soil liquefaction. Six input parameters such as soil initial effective mean confining pressure, initial relative density after consolidation, percentage of fines content, coefficient of uniformity, mean grain size and coefficient of curvature, are considered to predict the strain energy density required for triggering liquefaction. Rigorous performance evaluation based on multiple statistical performance evaluation showed that GPR based method performs better in terms of fitting statistics and error measures with less bias, when compared with many empirical expressions and data driven based expressions reported in literature. The parametric sensitivity study showed that σ'_{mean} is the most influencing predictor for liquefaction triggering.

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